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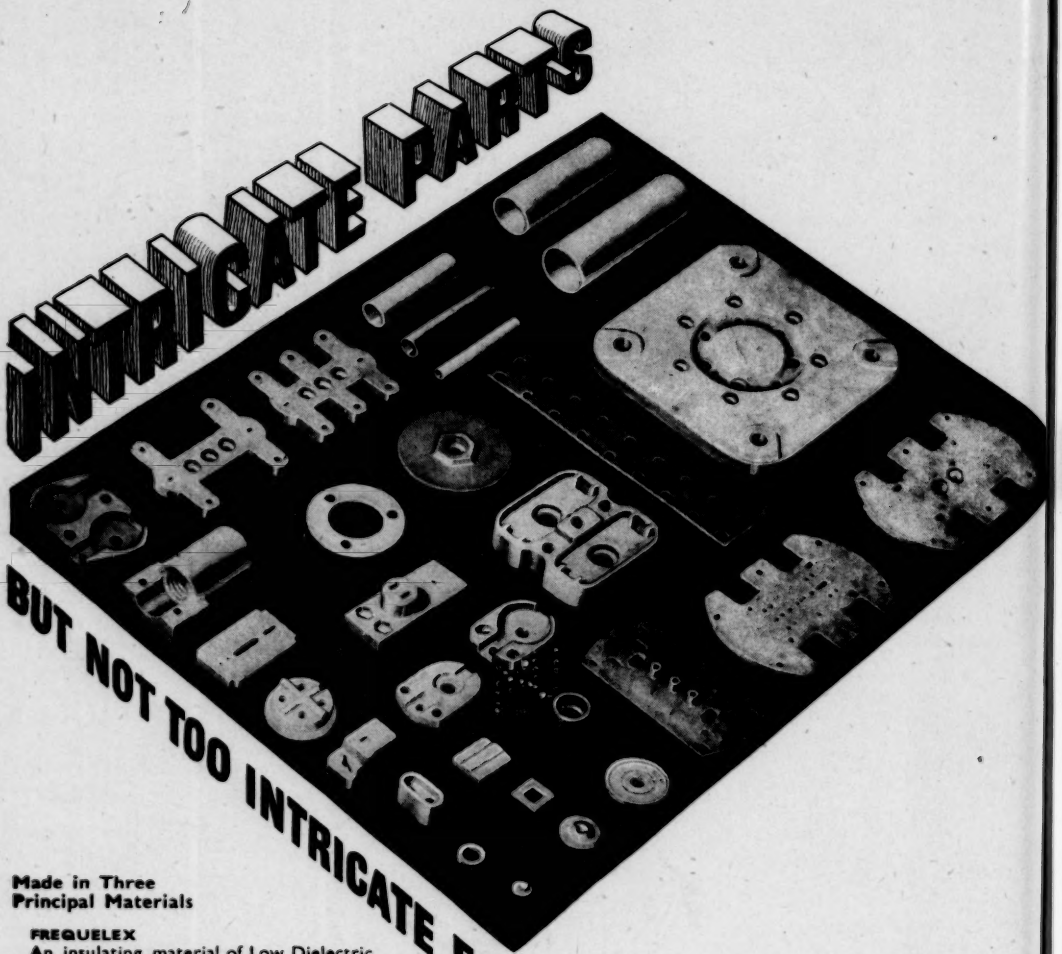

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EDITORIAL

Another Problem of Two Electrons

WHETHER as a result of our Editorial of March 1944, on "A problem of two electrons and Newton's third law" or not we do not know, but a number of somewhat similar paradoxes have recently appeared in *Electrical Engineering*, the journal of the American Institute of Electrical Engineers, under the title of "Electrical Essays for Recreation." As one correspondent says: "Material of this sort stimulates discussion and thinking about the fundamental principles of electrical engineering, and should therefore be highly welcome." Most of the examples given are of a relatively simple type, in which the apparent paradox is due to an easily detected false assumption, but one correspondent raises the time-honoured question of the two electrons moving adjacent to each other at high speed in a straight line, as in a beam of a cathode-ray tube." He says:* "These electrons are moving charges and therefore constitute currents. Hence, there should be an attractive force between them since they are adjacent currents having the same direction. (This is in addition to the normal electrostatic repulsion between like charges.) However, if the observer should travel along at the same speed as the electrons, they will appear to be at rest and he will conclude that for these stationary charges there is no attractive force. Obviously, whether or not this force exists must be entirely independent of what the observer does, so that some fallacy must exist in the above reasoning." We are pleased to note that this correspondent sticks to realisable conditions and pictures the electrons in the beam of a cathode-ray tube; they are usually stated to be moving through remote space, without any indication as to how they got there or where they are going.

What does one mean when one says that an electron is moving? There is only one reasonable answer to this question and that is, that the electron is moving relatively to the body, circuit, or system of which it forms an integral part. This is the velocity that is implied in any law involving electronic velocities and not the velocity relative to some acrobatic observer. When we say that the steady current in some d.c. transmission line is really a slow drift of electrons in the conducting material, it is, of course, open to anyone walking along the route in the same direction as the drift—he would have to move very slowly—to maintain that there was no drift of electrons, but that the copper wires, both go and return, were moving slowly in the opposite direction. He would also have to agree that there was a drift of electrons in the other wire at twice the velocity of the copper. You may say that this would be ridiculous, but would it be any more ridiculous than to maintain that the electrons in the cathode-ray tube are at rest and that consequently the cathode and anode and all the rest of the system of which the electrons are an integral part are moving at a high velocity

* *Loc. cit.*, October 1945, p. 381.

in the opposite direction? One may retort that the result, so far as any attraction between the electrons is concerned, must be independent of the speed of the observer; this is true, but the expression of the laws governing the phenomena is not necessarily independent. Whether the electrons are flying through the tube from cathode to anode or returning leisurely through the external circuit, their velocity in any calculation or expression of laws is understood to be that relative to the system. This greatly simplifies matters because one is not concerned with any movements of the positive charges produced by the removal of the electrons.

These considerations led us to a further examination of the nature of the field surrounding a solitary electron moving along the axis of a cathode-ray tube.

In Fig. 1 an electron is assumed to have been emitted from a cathode somewhere to the right, to have arrived at the anode A at a high velocity and to have passed through the central aperture in the anode. It is shown still moving axially along a metal tubular extension of the anode. Fig. 1 (*a*) shows the electric field, and the charges on

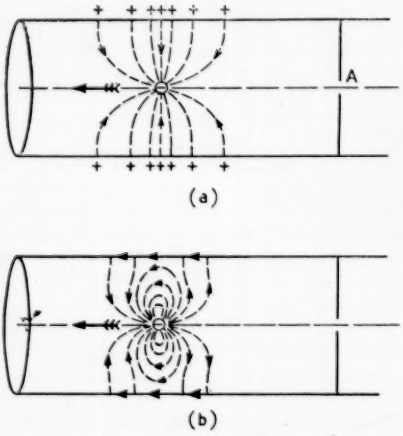


Fig. 1.

the tube; Fig. 1 (*b*) shows the distribution of electric currents—conduction currents in the metal and displacement currents in the vacuous space. This diagram is worth a little careful consideration; it should be compared with Fig. 5 of March 1944, in which the outer conductor was assumed to be far removed. Near the electron the displacement currents will still be much the same as before, but they are very much modified as the field departs more and more from the spherically radial distribution. At the surface of the metal tube the displacement currents must be normal to the surface, assuming the metal to be a perfect conductor. The total resultant current crossing any normal cross-section of the tube must be zero; if the plane passes through the electron, the conduction current at that point of the tube together with the sum of the displacement currents will just counterbalance the electronic current. At any other cross-section the sum of the conduction and displacement currents must be zero.

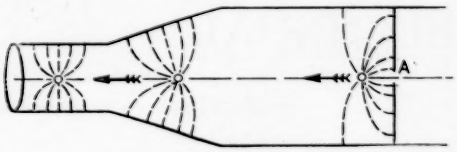


Fig. 2.

When the electron emerged from the aperture *A* it slowed down to some extent due to its electric field passing to the back of the anode and exerting a longitudinal retarding force on the electron. If the tube were now made conical as shown in Fig. 2 the electric field would exert an accelerating force on the electron which would therefore enter the smaller tube at an increased velocity. The displacement current density and therefor the energy per cubic centimetre due to the magnetic field in the neighbourhood of the electron would be increased, the latter varying as the square of the velocity. This would compensate for the reduction in the space occupied by the field, so that, as the diameter changed the speed would adjust itself so as to keep the total energy of the moving electron, that is, the energy of its magnetic and electric fields, constant. As the diameter of the tube decreased some of the electric field energy would be expended in accelerating the electron and thus converted into magnetic energy.

We have discussed this example in order to show how the ordinary expression of the electro-magnetic laws is based on certain assumptions. We would not relish the task of explaining the phenomena involved on the assumption that the electron was at rest and the apparatus flying through the air at a terrific but variable speed.

G. W. O. H.

BRIDGE CIRCUITS WITH A NON-LINEAR ELEMENT*

Valve-Voltmeter with a Stable Zero Adjustment

By M. Levy, A.M.I.E.E.

Summary.—If a uni-directional voltage is applied on one diagonal (supply input), a fraction appears on the second diagonal (output). Two methods are described which enable one to suppress this disturbance : the first method is to apply a convenient bias on the non-linear element ; and the second is to simulate the voltage-current characteristic of the non-linear element on one of the three other arms of the bridge. It is shown that this can be obtained by simple means such as a combination of a neon tube and some resistances.

When the bridge is balanced in this way, the output will be practically unaffected by any variation at the supply input and will depend only on the signals applied directly to the non-linear element.

By the application of the above methods, the usual zero fluctuations of valve-voltmeters are reduced to a small percentage of their usual value. This enables the sensitivity to be increased without increasing the zero fluctuations. Some circuits of very sensitive valve-voltmeters are described, and their average characteristics are as follows :—

Sensitivity : Full-scale deflection for a fraction of a volt.

Stability of zero : No appreciable variation of zero when the H.T. supply varies from 200 to 300 volts.

Stability of calibration : Less than two per cent. for the above H.T. supply voltage variation.

Detailed characteristics are given.

FIG. 1 represents a usual valve-voltmeter circuit. This is a bridge circuit in which the anode-cathode resistance of the valve forms one arm, the cathode resistance R_c a second arm and the resistances R_1 and R_2 the two other arms. The H.T. voltage is applied across one diagonal and the meter across the second one. The detected voltage to be measured is applied between grid and earth. The elements of the bridge are adjusted so that the meter gives no deflection (or full-scale deflection) when there is no signal. When a signal is applied on the grid, the anode-cathode resistance varies, the bridge is unbalanced and a deflection appears on the meter.

The sensitivity of this circuit is usually limited by the fluctuations of the needle around the zero in the absence of a signal. These fluctuations may be produced,—

(*a*) by variations of the characteristics of the valve,

(*b*) by variations of the heater voltage,

(*c*) by variations of the H.T. supply voltage.

It is well known that variations of the characteristics of the valve can be greatly reduced by negative feed-back. In practice, these variations are slow and of secondary importance. Variations of the heater voltage do not produce great fluctuations as the anode current is practically independent of the heater voltage within large limits. The zero fluctuations are mostly produced by

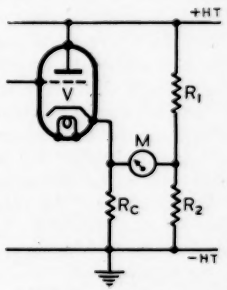


Fig. 1. *Usual bridge circuit for valve voltmeters.*

variations of the H.T. supply voltage. It is well known that the anode-cathode resistance of the valve does not behave as a real resistance, that is to say the current is not proportional to the voltage. If the bridge is balanced for one value of the H.T. supply voltage, this balance will not be maintained for other values of this voltage. In practical circuits a variation of 10 per cent. of the H.T. voltage may produce a deflection of the meter of nearly 20 or 30 per cent. of the scale. Thus a stabilised H.T. supply voltage is necessary, especially in case of sensitive circuits.

This solution is efficient but not very practical in many applications. If the valve-voltmeter is used as an independent unit, a stabilised H.T. supply can be obtained either with a saturated coil or with a neon-

* MS. accepted by the Editor, July 1945.

stabiliser circuit. In the first case the power unit is very large for the very small power supplied and produces a strong radiation field ; in the second case the power supplied to the neon-stabiliser circuit is high compared to the power feeding the bridge circuit. If the valve-voltmeter is part of a circuit, one has either to stabilise the voltage supplying the valve-voltmeter only or to stabilise the whole H.T. supply. In the first case the solution may not be practical for the reasons mentioned above and in the second case for evident reasons.

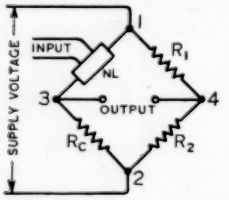


Fig. 2. *Bridge circuit with a non-linear element.*

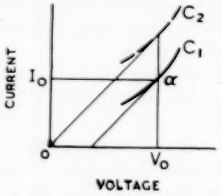


Fig. 3. *Characteristic of the element of Fig 2 ($NL + R_c$).*

The present method makes the bridge circuit practically independent of H.T. supply voltage variations and this voltage therefore need not be stabilised. The method is either to apply a convenient bias to the grid of the valve in order to render the anode current proportional to the anode voltage or to replace one of the resistances of the bridge by a convenient circuit having the same voltage-current characteristic as the valve. In this way the meter becomes insensitive to H.T. variations and sensitive only to voltages applied to the grid of the valve.

The method can be applied to any bridge circuit comprising a non-linear element and will be explained in detail for this general case.

Fig. 2 represents a bridge circuit composed of a non-linear element NL and three resistances R_1 R_2 R_c. A supply voltage is applied across diagonal 1–2 and a signal to be amplified, detected or measured, is applied on the non-linear element. The output is taken across diagonal 3–4. Assume that in the absence of an input signal the bridge is balanced for a given supply voltage V_0. If the supply voltage is varied, the balance is destroyed and an output voltage, or current, will appear. This is due to the non-linear voltage–current characteristic of the $NL + R_c$ branch. Referring to this characteristic represented in curve C_1 Fig. 3, let α be the actual working point corresponding to a supply voltage V_0. If the variations of V_0 are small, one can replace curve C_1 by its tangent at point α. This tangent usually does not pass through point O, the origin of the axis, and the variation of current across $(NL + R_c)$ is not proportional to the variation of voltage. For this reason if V_0 is varied, the bridge is no longer balanced. However, it is clear that this disturbance can be avoided either by displacing the characteristic from C_1 to C_2, in order that the new tangent corresponding to V_0 may pass through the origin, or by replacing resistance R_1 or R_c by an element having the same voltage-current characteristic as NL.

The way of applying the first method depends on the nature of the non-linear element NL. If it is a valve, the displacement can be obtained by a convenient grid bias as will be explained.

One way of applying the second method is well-known : if R_1 or R_c is replaced by a similar element to NL, the balance is obtained. However, this may not be desirable if the element NL is expensive or has big dimensions or other inconveniences. If NL is a valve R_1 or R_c can be replaced by the same type of valve. However, this solution is not very practical because it increases the dimensions, the wiring and the cost of the circuit. Besides, it may be shown that the sensitivity is then considerably reduced. By means of a neon tube and resistance circuit elements can be produced which have the same characteristics as NL and

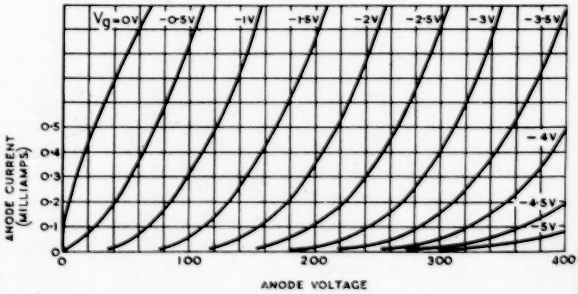


Fig. 4. *Characteristics of a triode. When the grid voltage varies the characteristic is displaced horizontally.*

which have not the drawbacks mentioned above.

Examples of Application of the First Method

Consider the valve-voltmeter circuit of Fig. 1. Fig. 4 represents the current–voltage characteristics of the valve for different grid-bias voltages. It may be observed that a change of grid-bias voltage produces a translation of the characteristic parallel to the voltage axis. From these curves one can deduce the current–voltage characteristic of the valve in series with a resistance R_c. This is represented in Fig. 5 for a grid bias equal to zero and each curve corresponds to a particular value of R_c. In order to give some numerical values, the curves have been traced for a triode Type 75. All these curves are practically straight lines nearly through the same point. It may be shown

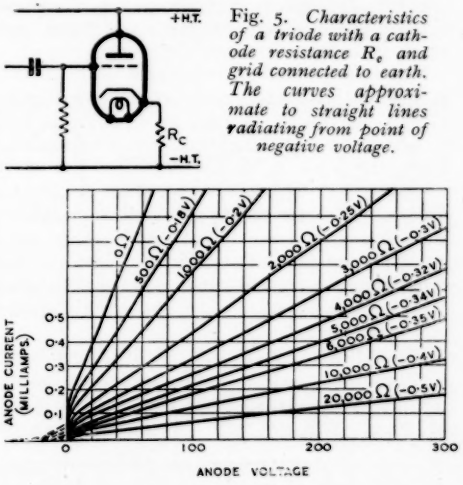


Fig. 5. *Characteristics of a triode with a cathode resistance R_c and grid connected to earth. The curves approximate to straight lines radiating from point of negative voltage.*

that the effect of applying a grid-bias voltage is to displace these curves by a translation parallel to the voltage axis exactly as for the curves of Fig. 4. A negative bias displaces the curves to the right. Thus it may be seen that for each value of R_c a convenient bias will make the corresponding characteristic pass through the original O. This value has been indicated on each line. It varies from 0 to nearly 0.5 volt.

Fig. 6 represents an application of this idea to a complete valve voltmeter circuit. The alternating voltage to be detected and measured is applied across terminals 1, 2. The diode D in connection with the filter R_bC detects this voltage and produces a negative voltage across condenser C. This voltage is applied to the grid of valve V and

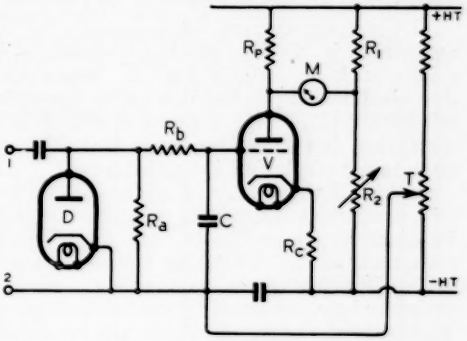
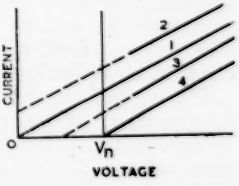


Fig. 6. *Valve-voltmeter circuit in which the grid is biased by means of potentiometer T.*

is measured on the meter M. To make the zero adjustment of the meter independent of H.T. voltage variations, the grid of this valve must be biased correctly. Note that even in the absence of a signal at terminals 1, 2, there is a small diode current which produces a drop of potential across resistance R_a. This drop can be used as bias if it has the right value, but usually, for high values of R_a, this drop is of the order of 1 volt and the negative bias on the grid of valve V is too great unless the cathode of the diode is connected to a positive bias of the order of a fraction of a volt. This can be done by connecting the cathode of the diode to the tapping T of a potential divider connected between H.T. and earth. *The zero adjustment is carried out by adjustment of R_2, and the stability of zero by adjusting the tap T.*

Fig. 7. *Voltage circuit characteristics. Curve 1 of a simple resistance; curve 2 for circuit of Fig. 8; curve 3 for circuit of Fig. 9 (b); curve 4 for circuit of Fig. 9 (a).*



Although this circuit gives good results its scope is limited because once the H.T. voltage and the sensitivity (determined by R_c) are chosen, the anode current of the valve is fixed and in some cases may be too small. By the methods which will be now

described the same stability effects can be obtained for any value of sensitivity, H.T. voltage *and anode current.*

Second Method and Examples of Application

The voltage–current characteristic of a resistance is a line passing through the origin, such as curve 1 Fig. 7. Means are described for displacing this curve upwards or downwards. Fig. 8 shows a circuit for displacing the curve upwards and Fig. 9 circuits for displacing it downwards.

In Fig. 8 the H.T. voltage is applied to two resistances R_1 and R_2 in series. It will be assumed for simplicity that R_2 is small compared to R_1. Then the current flowing in R_1 is proportional to the H.T. voltage and may be represented by curve 1 of Fig. 7. The current flowing in R_2 is the resultant of the current flowing in R_1 and the current coming from the neon tube N_1 through resistance R_4. It will also be assumed for simplicity that the voltage across R_2 is small compared with the voltage across the neon tube, so that resistance R_4 is large compared with R_2. Then the current flowing in R_2 and coming from the neon tube is constant

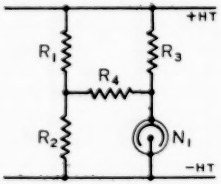


Fig. 8. *Circuit giving in resistance R_2 a current following the law of curve 2 of Fig. 7.*

and practically independent of the H.T. voltage. The current flowing in R_2 is thus the sum of a current proportional to the H.T. voltage and a current of constant value. It may be represented by curve 2 of Fig. 7, which is obtained by displacing curve 1 vertically by an amount equal to the constant current coming from the neon tube. The value of this displacement can be changed by adjusting R_4.

Fig. 9 gives circuits which displace curve 1 downwards. In Fig. 9 (*a*) the H.T. voltage is applied to a resistance R_2 through the neon tube N_2. The current in R_2 is proportional to the voltage applied to it, that is to say to the H.T. voltage reduced by V_N volts, V_N being the constant voltage across the neon tube. Thus this current may be represented by curve 4 in Fig. 7. The limitation of this circuit is due to the fact that curve 4 meets the voltage axis at the fixed point V_N and cannot be displaced. This can be done with the circuit of Fig. 9 (*b*) which is similar to the one of Fig. 8 except that the neon tube is connected to the + H.T. instead of the — H.T. terminal. This produces a displacement downwards of curve 1 (curve 3, Fig. 7) instead of upwards.

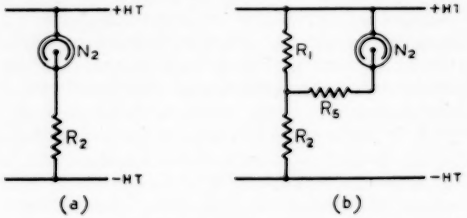


Fig. 9. *Circuit (a) gives in R_2 a current following the law of curve 4, Fig. 7, and (b) one corresponding to curve 3*

Some examples of application of these circuits to valve voltmeter circuits are given in Figs. 10, 11 and 12.

Fig. 10 represents a valve voltmeter circuit in which the grid is connected to earth by a high resistance R_g and in which the bias is obtained by a cathode resistance R_c. Thus, the $V_a - I_a$ characteristic of the valve is of the type shown in Fig. 5 or in curve 2 Fig. 7. The current I_a flows through resistance R_c and in order to get a zero adjustment independent of H.T. voltage variations, the same type of current must flow in resistance R_2. To obtain this result the circuit shown in Fig. 8 is used.

The adjustment of this circuit is very easy. First one must obtain in R_2 a current having the required slope in order that any variation of H.T. voltage will produce the same

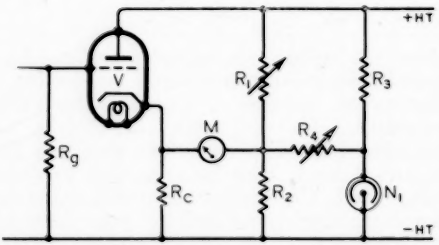


Fig. 10. *Bridge circuit of Fig. 1 combined with circuit of Fig. 8.*

voltage variation in R_c and R_2. This slope is determined by curve 1 (Fig. 7) and can be adjusted by varying resistance R_1. Thus one has first to adjust R_1 in order that the needle

of the meter will not move when the H.T. voltage is varied. This will be obtained usually with a certain deflection of the needle. This deflection will then be reduced to zero by adjusting resistance R_4, that is to say by sending a convenient current through R_2 which remains constant when the H.T. voltage is varied.

A circuit of this type has been tested, using the following values for the components :—Valve = triode Type 75 ; Meter = 200 μA full scale deflection ; R_c = 2,000 Ω ; R_1 = 63,000 Ω ; R_2 = 400 Ω ; R_3 = 55,000 Ω ; R_4 = 100,000 Ω ; Neon tube = S.T.C. Type VLS.405.

The following results were obtained :—

Sensitivity : Full scale deflection for — 0.25 volt applied to the grid.

Zero Stability : $\pm$ 0.3 μA when the H.T. voltage is varied from 200 to 300 volts; $\pm$ 2 μA ($\pm$ 1 per cent. of full-scale deflection) when the heater voltage is varied by $\pm$ 10 per cent.

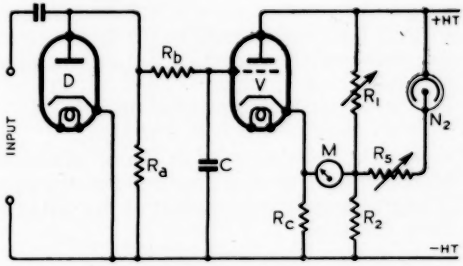


Fig. 11. *Bridge circuit of Fig. 1 combined with a diode detector and Fig. 9 (b).*

Fig. 11 represents the same valve voltmeter as in Fig. 10 except that the grid is connected to a diode detector as in Fig. 6. As has already been explained, the diode circuit introduces a negative bias on the grid of the order of 1 volt. This displaces the characteristics of Fig. 5 to the right so that one obtains a family of curves crossing the voltage axis on the right-hand side of the origin O of the axis. These lines are thus of the type represented by curve 4 in Fig. 7 and the circuit of Fig. 9 (*b*) must be used. The circuit of Fig. 11 gives the same stability as the circuit of Fig. 10. The circuit tested had the following values of elements :—Diode = Mullard EA50 ; Triode = Type 75 ; Meter = 200 μA full scale ; Neon tube = S.T.C. type VLS.405 ; R_a = R_b = 2 MΩ ; C = 0.01 μF. ; R_c = 2,000 Ω ; R_1 = 93,000 Ω ; R_2 = 400 Ω ; R_5 = 180,000 Ω.

To obtain a stable zero one must adjust resistances R_1 and R_5. Unfortunately this adjustment must be simultaneous and is more elaborate than in the case of Fig. 10. To use the circuit of Fig. 10 with a diode

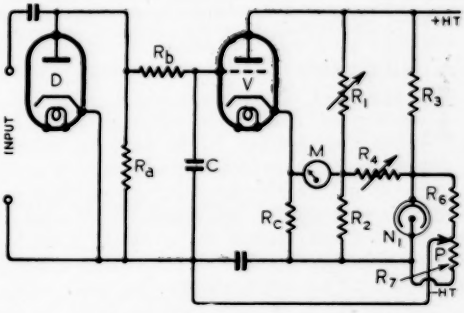


Fig. 12. *Diode detector combined with bridge circuit of Fig. 1 and balancing circuit of Fig. 8.*

detector circuit, one must bias the grid of the triode positively. This can be done easily by connecting the cathode of the diode to the moving tap of a potential divider connected across the neon tube N_1. In this way the circuit of Fig. 12 is obtained.

A valve voltmeter of this type has been tested. The values of the different elements were as follows :—Diode = Mullard EA50 ; Triode = Type 75 ; Neon tube = S.T.C. type VLS.405 ; Meter = 200 μA full scale deflection ; R_a = 10 MΩ ; R_b = 2 MΩ ; C = .01 μF. ; R_c = 10,000 Ω ; R_1 = 0.23 MΩ ; R_2 = 2,000 Ω ; R_3 = 70,000 Ω ; R_4 = 27,000 Ω ; R_6 = 1 MΩ ; R_7 = 0.1 MΩ approximately.

The characteristics of this circuit were found to be :—

Sensitivity : Full scale deflection for 0.6 volt A.C. applied on the diode.

Zero Stability : Less than 2 μA variation when the H.T. voltage varies from 200 to 300 volts (1 per cent. of full-scale deflection).

Calibration : Less than 2 per cent. variation of sensitivity at full scale when the H.T. voltage is varied from 200 to 300 volts.

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POWER LOSS IN ELECTROMAGNETIC SCREENS*

By C. F. Davidson, R. C. Looser and J. C. Simmonds

SUMMARY.—A method is developed which enables the increase in resistance of an inductor to be calculated when a screen consisting of circular loops of wire is brought near it. The calculated resistance increase agrees within about 20 per cent. or better with experimental values obtained by measurement on small screens.

1. Introduction

IN the past electromagnetic screens consisting of concentric circular loops of copper wire have been used to reduce power losses in nearby objects due to eddy currents induced by large inductors. Screens of this description are relatively simple to erect but no means have been available to enable the number of loops and spacing necessary to reduce the induced eddy currents to a certain relative level to be found. Neither has it been possible, except by rather crude methods, to determine the power loss in the screen. In this paper a method of calculating the power loss in the screen is developed. From this calculated power loss and measurements on the inductor, the effectiveness of the screen can be determined. The dimensions of the systems investigated are assumed small compared with one wavelength at the operating frequency. The rationalised MKS system of units is employed throughout.

From measurements made at different points behind the screen the efficiency of the screen at these points was found. Measurements were also made to determine the effect of numbers of loops upon the efficiency of the screen.† The results are shown in Figs. 1, 2 and 3, in which the figures shown on the curves give the distance of the pick-up coil behind the plane of the screen; all screen wires were of No. 20 S.W.G. tinned copper wire. The energising coil consisted of 55 turns of 26 S.W.G. wire on a 2-inch diameter former. The turns occupied a length of 1.2 inches and the coil was situated so that the nearest turn was 2 inches from the plane of the screen. For convenience the energising voltage applied to the inductor was at a frequency of 65 kc/s.

From these results it is seen that :—

(1) The efficiency of a large diameter screen, consisting of a large number of loops, is approximately independent of the distance from the axis.

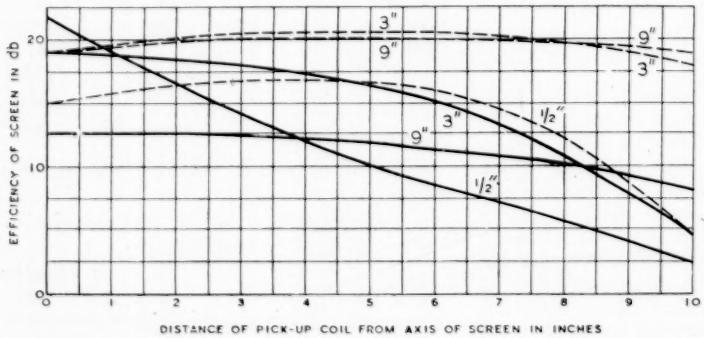

Fig. 1. *These curves of measured screen efficiency are (solid lines) for a screen of 25 concentric loops ranging from ⅜in. to 11in. diameter and approximately equally spaced. The dotted curves are for a screen of 10 concentric loops with diameters ranging from 1in. to 11in., and then 9 loops up to 20in. in diameter all equally spaced.*

2. Preliminary Measurements

An energising inductor was set up in front of a screen of circular loops of wire, and the voltage induced in a small search coil placed behind the screen was measured.

(2) Up to a point the efficiency increases with distance from the screen.

* MS. accepted by the Editor, August 1945.

† Screen efficiency is here defined as the logarithmic ratio of the field intensity which would exist at a point behind the screen if the screen were not in position, to the field intensity at the point when the screen is in position.

(3) The efficiency increases with the number of loops.

(4) Loops near the edge of the screen affect the efficiency near the axis.

It was also observed that the direction of the field behind the screen was roughly normal to the plane of the screen even at distances behind the screen approaching half its diameter. This fact would simplify the calculation of the power loss in a homogeneous material placed behind the screen, as, for example, in the case of an earth screen.

mutual inductance between every loop and every other loop, and the mutual inductance between every loop and the inductor must be calculated; a system of simultaneous equations has then to be solved, the number of unknowns and equations being equal to the number of loops. As a screen will rarely consist of less than 10 loops the computational difficulties are readily appreciated. A method of estimating the loop currents must, therefore, be sought.

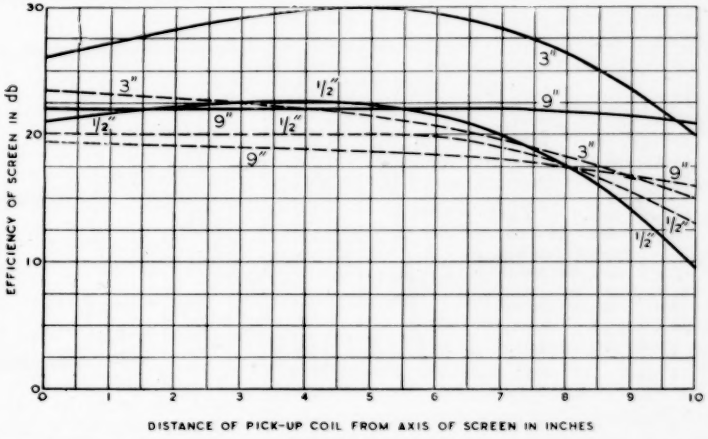

Fig. 2 (Right). *These curves are similar to those of Fig. 1, but the full line curves are for 20 loops of 1in. to 11in. diameter and then 9 loops to 20in. diameter. The dotted curves are for a screen of 20 loops of 1in. to 11in. diameter and then 5 loops only with diameters up to 20in.*

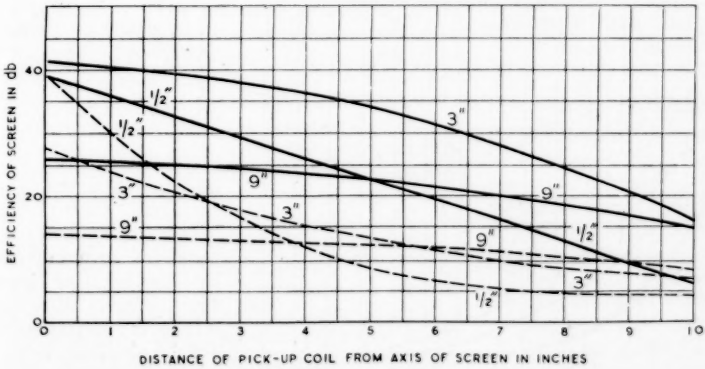

Fig. 3 (Left). *The full-line curves here represent one copper ring of* No. 18 *S.W.G. of 1in. internal diameter and 11in. external diameter and then 9 loops equally spaced to 20in. diameter. The dotted curve shows the results without these last 9 loops.*

3. Calculation of Power Loss and Screen Efficiency

If we could determine the currents induced in the loops of the screen then the power loss in the screen and the efficiency of the screen could be found. It is, however, impracticable to calculate the loop currents when the number of loops is more than four or five—the inductance of every loop, the

3.1. *Estimation of loop currents*

The eddy current surface density induced in a thin plane of conducting material by a circular loop carrying a current $I \cos \omega t$ placed at distance a from the plane and parallel to it is given by‡ :—

‡ See Appendix and Fig. 6.

$$i_\phi = \frac{I \cos \omega t \times k(a/b)}{4\pi\,(\rho/b)^{3/2} \times b} \left\{ -2K + \frac{2-k^2}{1-k^2} E \right\} \quad .. \quad .. \quad (1)$$

where $k^2 = \dfrac{4\,\rho/b}{[(1+\rho/b)^2 + (a/b)^2]}$

b = radius of exciting loop

ρ = radius vector in plane

K and E = complete elliptic integrals of the first and second kind respectively and modulus k

ω = $2\pi \times$ frequency

t = time

to the current flowing in a conducting plane over a radial distance $\rho - \dfrac{d}{2}$ to $\rho + \dfrac{d}{2}$ when the currents are induced by the same exciting loop. This assumption will be more justified as the relative distance between loops decreases and as the extent of the screen increases.

In Figs. 4 and 5 the term $(i_\phi b/I)$ is shown graphically against (ρ/b) for various values of (a/b). From these curves the current in any loop of a screen can be found per ampere in the exciting loop by reading off the value of the term and multiplying by (d/b). The loop current for any current in the exciting loop then follows directly, whilst the loop current for an exciting inductor consisting of a number of loops can be found by adding the loop currents which would be produced by each exciting loop separately.

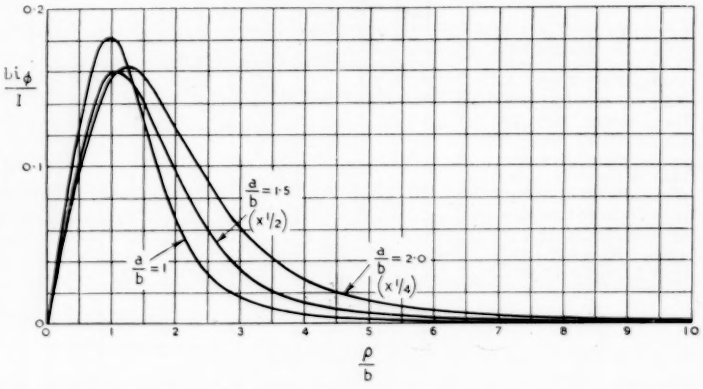

Fig. 4 (Right). *The variation of current density in a plane sheet with distance from the axis of the energising coil is shown here. The curves are for the three different values of a/b and the value read from the curve must be multiplied by the appropriate factor to give bi_ϕ/I.*

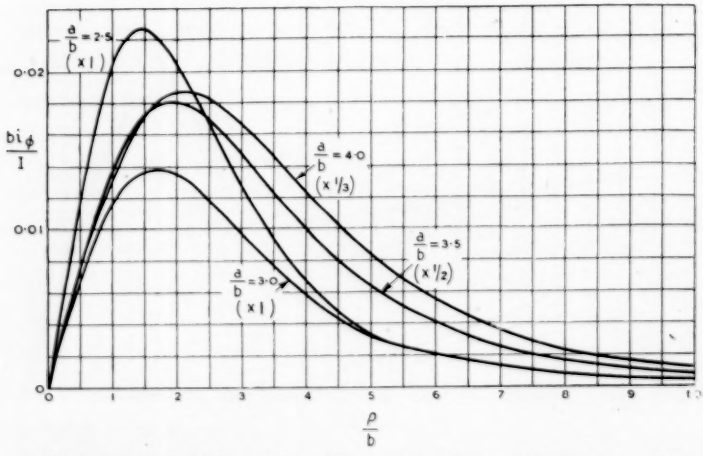

Fig. 5 (Left). *These curves are similar to those of Fig. 4, but for four further values of a/b.*

The currents induced in the conducting plane flow in circles concentric with the axis of the exciting loop. It is not unreasonable to assume that the current carried by a loop of radius ρ in a screen consisting of circular loops d apart, will be approximately equal

3.2. *Increase in resistance due to the presence of the screen*

Once the loop currents are found, the power loss in each loop is simply the product

of the square of the loop current and the loop resistance, and the total power loss is obtained by summing all the loop losses. The increase in inductor apparent resistance due to the screen is then found by dividing the total screen power loss by the square of the inductor current, or,

$$\Delta R = \frac{1}{I^2} \sum_{n=1}^{n=N} i^2_{\phi n}\, d^2\, R_n,$$

where ΔR = increase in resistance
N = number of loops in screen
R_n = resistance of nth loop

3.3. *The magnetic field produced by the screen currents*

Formulae which enable the magnetic intensity, due to a current flowing in a circular loop, at any point in space to be calculated are available. These formulae, however, are not of a simple form and a fair amount of computational work is involved. The magnetic intensity on the axis of the loop is given, however, by the relatively simple expression :—

$$H = \frac{I}{2\rho\,[1 + (a/\rho)^2]^{3/2}}$$

From this expression the intensity at any point on the axis distant a from the loop can be calculated providing the loop current I is known. The intensity produced by the whole of the loops of a screen can obviously be found by addition. The resultant intensity behind the screen will be the difference between the intensity produced by the screen currents and the intensity produced by the exciting inductor in the absence of the screen, whilst the screen efficiency at the point under consideration is the logarithmic ratio of the resultant field to the field produced by the exciting inductor in the absence of the screen. It will be shown below that, for screens of the type likely to be encountered in practice, the screen efficiencies calculated in this way are quite unreliable, due partly to the fact that the difference between two almost equal magnetic intensities is involved, each of which must be known with a high degree of accuracy.

4. Experimental and Calculated Results

Measurements were made on a number of small screens to justify, if possible, the proposals made in sections 3.2 and 3.3. That is, in effect, to determine to what degree the fundamental assumption made in section 3.1 is justified.

4.1. *Evaluation of resistance increase*

The increase in resistance of an exciting inductor when various loop screens were brought near it was measured at a frequency of 20 kc/s. Owing to the very small resistance change in some cases the measurement accuracy was, in these cases, quite poor. Details of the screens, the calculated resistance increase, the measured resistance increase, and the possible measurement error are given in Table I. A table showing the stages in the calculation of the resistance increase for a particular screen is given in Table II. This table shows very clearly the important parts of the screen from the point of view of power loss—the larger part of the power loss occurs in those loops which are about the same diameter as the exciting inductor. Loops of radius 1 in. to 5in. in this particular screen, Screen 4 in Table I, were replaced by loops of heavier gauge wire converting it to Screen 5 in Table I. The reduction in resistance increase is marked and shows the importance of these loops. Except in those cases (which would not normally arise in practice) where the screen had very few loops or was very near to the exciting inductor, the calculated and measured resistance increases do not disagree by more than 20 per cent. and no doubt some of the difference is due to measurement error.

4.2. *Screen efficiency*

For a point on the axis of, and just behind the screen, the field strengths due to the exciting inductor and to the currents in the loops of the screen were calculated, and the resultant field strength deduced, by the method given in section 3.3. The resultant field strength was also measured. Table III shows the calculated and the measured field strengths ; the agreement is very poor indeed, no doubt largely due to the fact that the difference of two almost equal fields is taken in the calculation. Thus, if the loop currents are in error by 10 per cent. say, the error on the resultant field may be very large indeed, whilst the error on power loss would only be 20 per cent.

5. Conclusions

(1) A method of determining the increase in resistance of an inductor due to the

presence of a loop screen has been developed and checked experimentally. The method should give the resistance increase to within 20 per cent. and, as the resistance increase due to the screen is usually made small compared with the inductor resistance, this degree of accuracy should be ample for most purposes.

(2) The method shows clearly those parts of the screen which contribute most to the power loss and enables steps to be taken to reduce the power loss most economically by increasing the gauge of the strategic loops or by placing them closer together.

(3) A similar method, when applied to the calculation of the screen efficiency, does not give results in agreement with experiment. This is because the difference of two almost equal quantities is involved in the calculation, hence the power loss in material placed behind the screen cannot be calculated directly. The total power loss due to the material of the screen and the material placed behind the screen may, however, be measured and since the former loss may be calculated, the loss due to the material behind the screen is known ; steps may then be taken to reduce this loss if necessary.

(4) Experimental results show that to obtain good efficiency behind the screen, even on the axis, the loops must be taken out to such a diameter that the currents in

TABLE I

Calculated and Measured Values for the Increase in Resistance of an Energising Coil due to the Presence of a Screen.

Screen No.	Details of Screens	No. of Turns and Diameter of Energising Coil	Increase in Resistance, Ohms				Possible Measurement Error, %
			Calculated		Measured		
			$\frac{a}{b}$	ΔR	$\frac{a}{b}$	ΔR	
1	12 screening loops, 1¼, 2, 3, 4¼ 6¼, 8¼, 10¼, 12¼, 14, 16, 18 and 20 ins. diameter.	55 Turns 2″ dia. ($b = 1''$)	1.0	0.273	1.0	0.190	1
			1.5	0.092	1.5	—	—
			2.0	0.039	2.0	0.033	1
			2.5	0.019	2.5	0.016	2
			3.0	0.0105	3.0	0.0090	4
			3.5	0.0062	3.5	0.0055	6
			4.0	0.0038	4.0	0.0035	10
2	24 screening loops, ¾, 1⅛, 1⅜, 1½ 2, 3, 3½, 4⅝, 5, 6¼, 7¼, 8, 9, 10¼, 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 ins. diameter.	55 Turns 2″ dia. ($b = 1''$)	1.0	0.166	1.0	0.151	1
			1.5	0.053	1.5	—	—
			2.0	0.022	2.0	0.024	1
			2.5	0.0103	2.5	0.0120	2
			3.0	0.0055	3.0	0.0055	4
			3.5	0.0032	3.5	0.0035	6
			4.0	0.0019	4.0	0.0020	10
3	10 screening loops, 4, 8, 12, 16, 20, 24, 28, 32, 36 and 40 ins. diameter.	110 Turns 4″ dia. ($b = 2''$)	1.0	4.58	1.0	1.258	1
			1.5	1.21	1.5	0.493	1
			2.0	0.454	2.0	0.204	1
			2.5	0.205	2.5	0.113	1
			3.0	0.104	3.0	0.067	2
			3.5	0.057	3.5	0.0385	3
			4.0	0.034	4.0	0.023	4
4	20 screening loops, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38 and 40 ins. diameter.	110 Turns 4″ dia. ($b = 2''$)	1.0	2.130	1.0	1.412	1
			1.5	0.593	1.5	0.495	1
			2.0	0.239	2.0	0.198	1
			2.5	0.102	2.5	0.094	1
			3.0	0.052	3.0	0.047	2
			3.5	0.029	3.5	0.027	3
			4.0	0.016	4.0	0.017	4
5	20 screening loops, as for screen 4, but the 2, 4, 6, 8 and 10 in. dia. loops were replaced by loops of 16 s.w.g. wire.	110 Turns 4″ dia. ($b = 2''$)	2.0	0.107	2.0	0.082	1
			3.0	0.033	3.0	0.032	2

All screening loops are of 20 s.w.g. tinned copper wire unless otherwise stated.

them are less than about 1/10th of the maximum loop current.

(5) If necessary the method of calculation could no doubt be extended to the case of a screen of loops which entirely enclose the inductor, by assuming the loops to lie upon the surface of a circular cylinder or a sphere.

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APPENDIX

The eddy currents induced in an infinite plane conducting sheet by a circular loop

Let a circular loop carrying a current $I \cos \omega t$ be placed with its plane parallel to a thin, infinite, plane, conducting sheet, as shown in Fig. 6. We wish to determine at any point in the sheet the eddy current density

TABLE II

Specimen Calculation for Determining the Increase in the Resistance of a Coil due to the Presence of a Screen

$a = 4''$, $b = 2''$, $\frac{a}{b} = 2$.

Radius of loop (ρ) ins.	$\frac{\rho}{b}$	$\frac{bi_\phi}{I}$ from Fig. 4	d, metres	Wire current (i) $= \left[\frac{bi_\phi}{I}\right] \times \frac{d}{b}$ amps.	Loop resistance, (R) ohms	Power loss in loop, $i^2R \times 10^6$ micro-watts
1	0.5	0.0241	0.0254	0.01205	0.0042	0.61
2	1.0	0.0388	0.0254	0.01940	0.0084	3.16
3	1.5	0.0390	0.0254	0.01950	0.0126	4.79
4	2.0	0.0314	0.0254	0.01570	0.0168	4.14
5	2.5	0.0226	0.0254	0.01130	0.0210	2.68
6	3.0	0.0154	0.0254	0.00769	0.0252	1.49
7	3.5	0.0103	0.0254	0.00518	0.0294	0.79
8	4.0	0.0070	0.0254	0.00350	0.0336	0.41
9	4.5	0.0050	0.0254	0.00251	0.0378	0.24
10	5.0	0.0034	0.0254	0.00171	0.0420	0.12
11	5.5	0.0025	0.0254	0.00127	0.0462	0.07
12	6.0	0.0018	0.0254	0.00092	0.0504	0.04
13	6.5	0.0014	0.0254	0.00072	0.0546	0.03
14	7.0	0.0010	0.0254	0.00052	0.0588	0.02
15	7.5	0.0008	0.0254	0.00040	0.0630	0.01
16	8.0	0.0006	0.0254	0.00032	0.0672	0.01
17	8.5	0.0005	0.0254	0.00028	0.0714	0.01
18	9.0	0.0004	0.0254	0.00020	0.0756	0.00
19	9.5	0.00025	0.0254	0.00016	0.0798	0.00
20	10.0	0.00025	0.0254	0.00016	0.0840	0.00
						19.73

In the above calculation a current of 1 ampere has been assumed in the energising coil.

From Section 3.2 $\Delta R = \frac{1}{I^2} \sum_{n=1}^{n=N} i^2 \phi_n d^2 R_n$

Hence ΔR for one turn $= 1.973 \times 10^{-5}$ ohm.
and ΔR for a coil of 110 turns $= 1.973 \times 10^{-5} \times (110)^2$ ohm.
$= \underline{\underline{0.239 \text{ ohm.}}}$

TABLE III

Calculated and Measured Values of the Field Strength $\frac{3}{8}$ in. behind a Concentric Screen.

Screen No.		Field Strength for (M.K.S. Units)		
		$\frac{a}{b} = 2$	$\frac{a}{b} = 2.5$	$\frac{a}{b} = 4$
1	Calculated	1.8×10^{-2}	1.3×10^{-2}	4.0×10^{-3}
See Table I	Measured	5.6×10^{-1}	2.8×10^{-1}	7.1×10^{-2}
2	Calculated	1.7×10^{-2}	4.0×10^{-3}	0.00
See Table I	Measured	2.4×10^{-1}	1.3×10^{-1}	3.2×10^{-2}

induced by the current in the circular loop. In such circumstances it has been shown by Clerk Maxwell that if the vector potential of the field produced by the source is

$$A' = f(t, x, y, z) \quad .. \quad .. \quad .. \quad (1)$$

i.e., if the vector potential is some arbitrary function of the co-ordinates of the point under consideration and of the time t then at time t the total vector potential of the eddy currents is

$$A = -\int_0^\infty \frac{\partial}{\partial t} f\left(t - \tau, x, y, z + \frac{2s}{\mu}\tau\right) d\tau \quad .. \quad .. \quad (2)$$

where s = area resistivity
τ = a time variable.

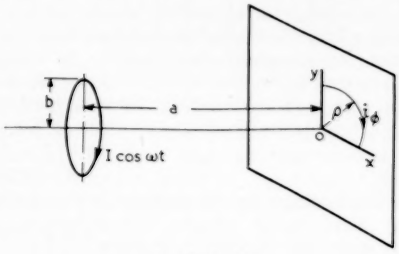

Fig. 6.

This formula depends upon the receding image representation of the field and further information should be sought in Maxwell's treatise or in, for example, W.R. Smythe's treatise.*

If we take the origin of our co-ordinate system in the sheet as shown in the drawing, it is easily proved that the vector potential of the loop current is given by :—

$$A_\phi' = \frac{\mu b I}{\pi} \int_0^{\pi/2} \frac{(2\sin^2\theta - 1)\, d\theta}{[(b+\rho)^2 + (a-z)^2 - 4b\rho\sin^2\theta]^{\frac{1}{2}}} \cos\omega t \quad .. \quad .. \quad .. \quad .. \quad (3)$$

where $\theta = \left(\frac{\phi}{2} - \frac{\pi}{2}\right)$. The vector potential of the eddy currents is, therefore :—

$$A_\phi = \int_0^\infty \int_0^{\pi/2} \frac{\mu b I \omega \sin\omega(t-\tau) \times (2\sin^2\theta - 1)}{\pi[(b+\rho)^2 + (a - z - \frac{2s}{\mu}\tau)^2 - 4\, b\rho\sin^2\theta]^{\frac{1}{2}}} d\theta\, d\tau \quad .. \quad .. \quad .. \quad (4)$$

the order of the partial differentiation with respect to time and the integration with respect to θ being reversed. Integrating by parts with respect to τ leads to :—

$$A_\phi + A'_\phi = \frac{-2bIs}{\pi\omega} \sin\omega t \int_0^{\pi/2} \frac{(2\sin^2\theta - 1)(a-z)\, d\theta}{[(b+\rho)^2 + (a-z)^2 - 4b\rho\sin^2\theta]^{3/2}} \quad .. \quad .. \quad (5)$$

as an approximation at high frequencies.

Thus, in the sheet, where $z = 0$, the total vector potential is

$$A_\phi + A'_\phi = \frac{-2bIs}{\pi\omega} \sin\omega t \int_0^{\pi/2} \frac{a(2\sin^2\theta - 1)\, d\theta}{[(b+\rho)^2 + a^2 - 4b\rho\sin^2\theta]^{3/2}} \quad .. \quad .. \quad (6)$$

The electric intensity is equal to the rate of decrease of vector potential and also equal to the current density multiplied by the area resistance, therefore

$$i_\phi = -\frac{1}{s}\frac{d}{dt}(A_\phi + A_\phi')$$

and

$$i_\phi = \frac{2bI}{\pi} \cos\omega t \int_0^{\pi/2} \frac{a(2\sin^2\theta - 1)\, d\theta}{[(b+\rho)^2 + a^2 - 4b\rho\sin^2\theta]^{3/2}} \quad .. \quad .. \quad .. \quad .. \quad (7)$$

Now make the substitution

$$k^2 = \frac{4b\rho}{[(b+\rho)^2 + a^2]} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (8)$$

* Static and Dynamic Electricity.

then $$i_\phi = \frac{2}{\pi} abI \cos \omega t \left(\frac{k^2}{4b\rho}\right)^{3/2} \int_0^{\pi/2} \frac{(2 \sin^2 \theta - 1)\, d\theta}{(1 - k^2 \sin^2 \theta)^{3/2}} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (9)$$

It is not difficult to show that :—

$$\int_0^{\pi/2} \frac{2 \sin^2 \theta - 1}{(1 - k^2 \sin^2 \theta)^{3/2}}\, d\theta = \frac{1}{k^2}\left[-2K + \frac{k^2 - 2}{k^2 - 1} E\right] \quad .. \quad .. \quad .. \quad .. \quad (10)$$

so that we have finally for the eddy currents

$$i_\phi = \frac{1}{4\pi} I \cos \omega t \frac{k(a/b)}{(\rho/b)^{3/2} \times b}\left[-2K + \frac{2 - k^2}{1 - k^2} E\right] \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11)$$

where K and E = complete elliptic integrals of the first and second kind respectively and of modulus k.

Equation (11) is not suitable for computation when k is small but the more suitable approximation

$$i_\phi = \frac{3}{64} I \cos \omega t \frac{k^5(a/b)}{(\rho/b)^{3/2} \times b} \frac{\left[1 + k^2/4 + \frac{15k^4}{128} + \ldots\right]}{(1 - k)^2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (12)$$

is easily established by expanding the elliptic integrals.

NEW BOOKS

Elementary Electric-Circuit Theory

By R. H. FRAZIER. Pp. 434 + x. McGraw-Hill Book Company, 330, West 42nd Street, New York, 18, N.Y. Price $4.

The author of this book is Associate Professor of Electrical Engineering at Massachusetts Institute of Technology. His aim has been to provide a text book in which the fundamentals of electric-circuit theory are presented in a way suitable for students who will later specialise in either light or heavy-current engineering. This effort is a most laudable one, for many of the standard text-books show more than a bias towards one or other field. It is perhaps inevitable that the communications engineer should feel in reading the book that the author has dwelt overmuch on the power side.

The work opens with definitions and concepts which are clearly, though not rigorously, discussed. The methods of circuit analysis are then introduced in a chapter on resistance networks and the ideas extended to reactive elements. The chapter on complex algebra, in the heading of which the use of *simplex* may confuse the telecommunication engineer, is not excessively long. There is no reason why the student should not consult an algebra book if his mathematical equipment is inadequate. The author then proceeds to more advanced circuit theory, the theory of non-sinusoidal waves and polyphase networks and ends with a chapter on transients. An appendix contains a number of useful tables.

The excellent sections dealing with equivalent, dual and reciprocal networks call for special mention in a work of this kind. Only too often the relation between these forms is obscured rather than clarified. On the other hand the powerful methods of transient analysis based on the Fourier transform are not mentioned and attention is concentrated on the classical approach. The Fourier transform does not present difficulties which the undergraduate should not be able to surmount, and its attractive simplicity makes it a desirable approach if only for the promise of power which it offers when used for simple problems.

The book is produced in accordance with American economy standards and each pair of pages carries the chapter and paragraph number. The references are adequate and include a number of early original papers ; the work of Ohm, Coulomb, Ampere and Maxwell is not overlooked in the flood of their followers. Each chapter is followed by a number of questions and problems, though it must be noted that two of the circuits given on page 256 are incorrect. H. J.

Radio Valve Vade Mecum, 1945. By P. H. BRANS. Algemeene en Technische Boekhandel, Prins Leopoldstr. 28, Antwerpen (Borgt), Belgium.

This book of 208 pages is in four languages—Flemish, French, English and German—and gives, chiefly in tabular form, characteristics and base connections of a vary large number of British, American and Continental (including Russian) valves.

Back Issues

REQUESTS are being received for back issues of *Wireless Engineer* from countries which were under enemy occupation during the war. Readers who have copies dating back to 1940 for disposal are invited to communicate with the Publisher.

THE RECTIFICATION OF SIGNAL AND NOISE*

By V. J. Francis, B.Sc., A.R.C.S., F.Inst.P., A.M.I.E.E., and E. G. James, Ph.D., B.Sc.

(Communication from the Research Staff of the M.O. Valve Company, Limited, at The G.E.C. Research Laboratories, Wembley, England).

SUMMARY. — It is shown that the D.C. component of rectified noise, obtained with a square law triode, is not proportional to the mean square noise unless the valve is biased to cut-off. Unless this condition is fulfilled, errors as large as 3 db. may be obtained if the assumption of proportionality is made.

It is shown that a linear diode with a resistive load shunted by a large capacitor provides an accurate method of measuring noise. The mean rectified current with no applied signal is directly proportional to the R.M.S. noise input, the factor of proportionality being a function of the load resistance multiplied by the diode conductance. When both noise and signal are present, the ratio of the rectified current to R.M.S. noise input is a function of the signal/noise input ratio and the product of load resistance and diode conductance.

For the case of a diode operating on the exponential part of the characteristic, the rectified current is obtained for an input consisting of a mixture of signal and noise. Results are given for various values of signal input voltage and R.M.S. noise input. An indication is given of the type of transition to be expected between the exponential characteristic and the linear characteristic as the mean diode current increases.

LIST OF SYMBOLS.

v_1 = Instantaneous value of input voltage.
v_2 = Instantaneous value of output voltage.
$\sigma^2 = \overline{(v_1 - \bar{v}_1)^2}$
$\bar{v}_1$ = mean value of input voltage.
$\bar{v}_2$ = mean value of output voltage.
V_1 = cut-off voltage of triode (V_1 a negative voltage).
V_2 = bias on triode.
$V = -V_1 + V_2$
i = instantaneous current through valve.
$\bar{i}$ = mean current through valve.
R = bias resistance for diode.
c = amplitude of sinusoidal carrier.
$p = \bar{v}_2/\sigma$
g = diode conductance.
$r = \bar{v}_2/c$
$q = \frac{c}{\sigma} = \frac{p}{r}$
$\alpha = \frac{q(r-1)}{\sqrt{2}}$
$\beta = \frac{q(r+1)}{\sqrt{2}}$
$b_n = \int_\alpha^\beta \phi^n \exp\{-\phi^2\}\, d\phi$
R_n see equations (46).
A, b constants.
ω = $2\pi \times$ frequency of sinusoidal input.
a_n see equations (29).
I_n = see equations (31).
θ_n see equation (39).

* MS. accepted by the Editor, July 1945.

1. Introduction

IN accurate measurements of noise, the most widely used measuring instrument is the vacuo-thermo-junction, which gives a true measure of the mean square value. The overload capacity of a vacuo-thermo-junction is, however, small, and it would be very convenient in making noise measurements on high gain R.F. amplifiers, to replace the thermo-junction by a thermionic detector.

In the past various experimenters have used the triode as an " anode bend " detector for measuring noise. Others have used a diode detector, since such a detector is generally incorporated in the receiver.

When the triode as an " anode-bend " detector has been used, the usual practice has been to bias the valve so as to obtain a steady anode current of the order of 1 mA in the absence of any signal, and to take the increase in anode current with applied signal voltage as a measure of that voltage. There has been some doubt, however, as to how much error was involved when comparing signal and noise voltage with this detector.

The current/voltage characteristic of a normal diode is approximately linear for currents higher than about 100 micro-amperes, but for currents smaller than this the characteristic departs from linearity and becomes exponential. When a diode rectifier is used for the measurement of

noise or signal to noise ratio, one can assume a linear characteristic for large input voltages, but this assumption is not justified when the input voltage is small.

Since the analysis of the response of an exponential diode to signal plus noise does not present great difficulty, and the numerical work involved is much less than in the case of the linear diode, it was thought worth while to examine the exponential as well as the linear case.

2. Measurement of R.M.S. Noise using Square-Law Triode

We assume that the input voltage v_1 is pure noise; that is that it is derived from random voltage pulses, and that all the circuits involved up to the rectifier are linear. If these assumptions are made, it is shown in recent papers[1] that the distribution of v_1 is Gaussian, so that if $W(v_1)\,dv_1$ is the probability that v_1 lies between v_1 and $v_1 + dv_1$

$$W(v_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{v_1^2}{2\sigma^2}\right\} \quad .. \quad (1)$$

where $\sigma^2 = \overline{(v_1 - \bar{v}_1)^2}$ .. .. (2)

If we take the characteristic of the triode (see Figs. 1 and 2) as

$$\left.\begin{aligned} i &= a\,(v_1 - V_1 + V_2)^2 \\ &= a\,(v_1 + V)^2 \qquad v_1 > -V \\ i &= o \qquad\qquad\quad v_1 < -V \end{aligned}\right\} \quad .. \quad (3)$$

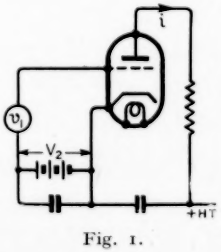

Fig. 1.

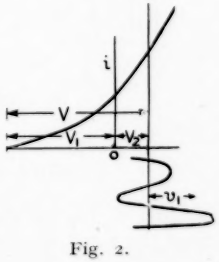

Fig. 2.

where a is a constant, and V is the bias voltage plus the cut-off voltage

$$\begin{aligned} \bar{\imath} &= a\,\overline{(v_1 + V)^2} \\ &= \int_{-v}^{\infty} a\,(v_1 + V)^2\, W(v_1)\, dv_1 \\ &= \frac{a}{\sqrt{\pi}} \int_{-v/\sigma\sqrt{2}}^{\infty} (V^2 + 2\sqrt{2}\,V\sigma y + 2\sigma^2 y^2) \\ &\qquad\qquad \exp(-y^2)\, dy \quad .. \quad (4) \end{aligned}$$

Using the identity

$$\int_x^{\infty} y^2 \exp(-y^2)\, dy = \frac{x}{2} \exp(-x^2) + \tfrac{1}{2}\int_x^{\infty} \exp(-y^2)\, dy \quad .. \quad .. \quad (5)$$

it is easy to show that

$$\frac{i}{a} = \frac{1}{\sqrt{\pi}} (V^2 + \sigma^2) \int_{-v/\sigma\sqrt{2}}^{\infty} \exp(-y^2)\, dy + \frac{V\sigma}{\sqrt{2\pi}} \exp\left(-\frac{V^2}{2\sigma^2}\right) \quad .. \quad (6)$$

Now, if $V_2 = V_1$ that is $V = 0$, this gives

$$i/a = \sigma^2/2 \quad .. \quad .. \quad .. \quad .. \quad (7)$$

which is clearly correct, since a square-law triode biassed to cut-off will give half the

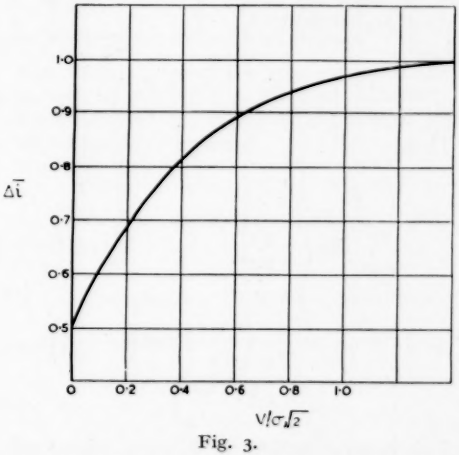

Fig. 3.

output of a perfect square-law rectifier such as a thermocouple. In general, however, the output will depend upon V.

If Δi is the increase in the output due to the noise, we have

$$\Delta\bar{\imath} = \bar{\imath} - aV^2 \quad .. \quad .. \quad .. \quad (8)$$

whence

$$\begin{aligned} \frac{\Delta\bar{\imath}}{a\sigma^2} &= \tfrac{1}{2}\left\{1 + \operatorname{erf}\left(\frac{V}{\sigma\sqrt{2}}\right)\right\} \\ &\quad - \frac{V^2}{2\sigma^2}\left\{1 - \operatorname{erf}\left(\frac{V}{\sigma\sqrt{2}}\right)\right\} \\ &\quad + \frac{1}{\sqrt{\pi}}\left(\frac{V}{\sigma\sqrt{2}}\right) \exp\left(-\frac{V^2}{2\sigma^2}\right) \\ &\qquad\qquad .. \quad .. \quad (9) \end{aligned}$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2)\,dy$ .. (10)

Fig. 3 shows $\Delta i/a\sigma^2$ plotted against $V/\sigma\sqrt{2}$ from which, if V is known and Δi is measured, σ can be found. It is clear that large errors can be made if (7) is assumed to be true for all values of V.

3. Measurement of R.M.S. Noise using Linear Diode

Consider the usual rectifier circuit shown in Fig. 4 where R is chosen small enough to allow the diode to operate on the substantially linear part of its characteristic and the capacitor is chosen so large that the output voltage v_2 is substantially constant and equal to $\bar{v}_2$.

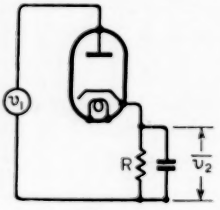

Fig. 4.

Let

$$\left.\begin{aligned} i &= g\,(v_1 - \bar{v}_2) \qquad v_1 > \bar{v}_2 \\ i &= o \qquad\qquad\quad v_1 < \bar{v}_2 \end{aligned}\right\} \quad .. \quad (11)$$

Then

$$\bar{\imath} = \frac{\bar{v}_2}{R} = \int_{\bar{v}_2}^{\infty} g\,(v_1 - \bar{v}_2)\,W(v_1)\,dv_1$$

$$= \frac{g\sigma}{\sqrt{2\pi}} \exp\left(-\frac{\bar{v}_2^2}{2\sigma^2}\right) - \frac{g\bar{v}_2}{2} + \frac{g\bar{v}_2}{2}\,\text{erf}\left(\frac{\bar{v}_2}{\sigma\sqrt{2}}\right) \quad .. \quad .. \quad (12)$$

$$1 + \frac{2}{Rg} = \frac{1}{\sqrt{\pi}}\left(\frac{\sigma\sqrt{2}}{\bar{v}_2}\right) \exp\left(-\frac{\bar{v}_2}{2\sigma^2}\right) + \text{erf}\left(\frac{\bar{v}_2}{\sigma\sqrt{2}}\right) \quad .. \quad .. \quad (13)$$

This is an equation for $\bar{v}_2/\sigma\sqrt{2}$, whose solution is of the form

$$\bar{v}_2/\sigma = f(Rg) \quad .. \quad .. \quad .. \quad (14)$$

so that whatever the value of Rg, $\bar{v}_2$ is proportional to σ. That is, for a self-biased linear diode, the mean rectified voltage or current, when the time-constant of the load is large, is strictly proportional to the R.M.S. value of the input noise. For $g = 2$ mA/V and $R = 1{,}000$ ohms.

$$\bar{v}_2 = 0.42\sigma \quad .. \quad .. \quad .. \quad (15)$$

4. Linear Diode Rectification of Noise-Modulated Carrier

4 (a). *The Amplitude Distribution Function.*

Consider the same circuit as Fig. 4, but now with a sinusoidal carrier of frequency $\omega/2\pi$ superimposed on the noise. The problem is to find the output voltage as a function of the carrier amplitude, c, and the R.M.S. noise, σ.

For this purpose we need to know the amplitude probability distribution for a noise modulated carrier. The probability that the noise amplitude, x, lies between x and $x + dx$ is :

$$W_1(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad (16)$$

The probability that the carrier amplitude, y, lies between y and $y + dy$ is easily shown to be :

$$W_2(y)\,dy = \frac{dy}{\pi\,(c^2 - y^2)^{\frac{1}{2}}} \quad .. \quad .. \quad (17)$$

Therefore, the probability that the noise amplitude lies between x and $x + dx$ and that the carrier amplitude lies between y and $y + dy$ is

$$W_1(x)\,W_2(y)\,dx\,dy = \frac{1}{\pi\sigma\sqrt{2\pi}} \cdot \frac{1}{(c^2 - y^2)^{\frac{1}{2}}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx\,dy \quad .. \quad .. \quad (18)$$

where the range of y is from $-c$ to $+c$, and the range of x from $-\infty$ to $+\infty$.

If we now make the transformation

$$\begin{aligned} x + y &= v_1 \\ y &= y \end{aligned}$$

we have

$$\int_{-\infty}^{+\infty} dv_1 \int_{-c}^{+c} W_3(v_1, y)\,dy = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} dv_1 \int_{-c}^{+c} \frac{1}{\sqrt{c^2 - y^2}} \exp\left\{-\frac{1}{2\sigma^2}(v_1 - y)^2\right\} dy \quad (19)$$

where $W_3(v_1, y)\,dv_1\,dy$ is the probability that the total amplitude lies between v_1 and $v_1 + dv_1$ and that the carrier component lies between y and $y + dy$.

Thus if $W_4(v_1)\,dv_1$ is the probability that the amplitude of the noise modulated carrier lies between v_1 and $v_1 + dv_1$ we have :

$$W_4(v_1) = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-c}^{+c} \frac{1}{\sqrt{c^2 - y^2}} \exp\left\{-\frac{(v_1 - y)^2}{2\sigma^2}\right\} dy \quad .. \quad (20)$$

The integral on the right of equation (20) considered as a function of v_1 with σ and c as parameters, fulfils the same rôle for a noise modulated carrier as the expression on the right of equation (16) as a function of v_1, with σ as parameter, fulfils for the noise voltage. This method of dealing with a noise modulated carrier may have other useful applications besides those considered in this paper. It should be noticed that the right-hand side of equation (20) is an improper integral, the integrand becoming infinite at the limits. It therefore might require careful handling in some cases, but the uses to which it is put in this paper do not lead to difficulty.

We thought that this method of dealing with combined signal and noise was new, but since this work was done we have found that the same method had been used by Bennett[2], although his analysis is restricted to the case of no bias on the detector.

4 (b). *The Equation for the Output.*

For a linear diode and the circuit of Fig. 4, we have, as before :

$$\left.\begin{aligned} i &= g\,(v_1 - \bar{v}_2) \qquad v_1 > \bar{v}_2 \\ i &= o \qquad\qquad\quad v_1 < \bar{v}_2 \end{aligned}\right\} \quad .. \quad (11)$$

$$\therefore\ \bar{\imath} = \frac{\bar{v}_2}{R} = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{\bar{v}_2}^{\infty} g(v_1 - \bar{v}_2)\, dv_1 \int_{-c}^{+c} \frac{1}{\sqrt{c^2 - y^2}} \exp\left\{ -\frac{1}{2\sigma^2}(v_1 - y)^2 \right\} dy \quad .. \quad (21)$$

Putting $y = cz$ and $v_1 = u\,\bar{v}_2$ this gives

$$\bar{v}_2 = \frac{Rg}{\pi\sigma\sqrt{2\pi}} \int_1^{\infty} (u-1)\, du \int_{-1}^{+1} \frac{1}{\sqrt{1 - z^2}} \exp\left\{ -\frac{1}{2\sigma^2}(u\bar{v}_2 - cz)^2 \right\} dz \quad (22)$$

And now putting

$$\bar{v}_2/\sigma = p\ ;\ \ \bar{v}_2/c = r\ ;\ \ c/\sigma = q = p/r$$

and for convenience changing the variables of integration we have

$$\int_1^{\infty} (y-1)\, dy \int_{-1}^{+1} \frac{1}{\sqrt{1 - x^2}} \exp\left\{ -\tfrac{1}{2}(yp - qx) \right\} dx = \frac{\pi\sqrt{2\pi}}{Rg\,p} \quad .. \quad .. \quad (23)$$

The solution of this equation is of the form

$$F\,(p, q) = Rg \quad .. \quad .. \quad .. \quad (24)$$

and therefore is the solution of our problem, since for given Rg we obtain p as a function of q or vice versa.

Equation (13) gives one limiting case of this equation, that is when $c = o$. The other limiting case, when $\sigma = o$ follows at once from equation (17), giving

$$\bar{v}_2 = \frac{Rg}{\pi} \int_{\bar{v}_2}^{+c} \frac{(x - \bar{v}_2)}{\sqrt{c^2 - x^2}}\, dx \quad .. \quad (25)$$

which leads to

$$r + \frac{Rgr}{2} - \frac{Rgr}{\pi} \sin^{-1} r - \frac{Rg}{\pi}\sqrt{1 - r^2} = 0 \quad .. \quad .. \quad (26)$$

For the particular value of $Rg = 2$, this gives $r = 0.33$, that is

$$\bar{v}_2 = 0.33c \quad .. \quad .. \quad .. \quad (27)$$

4 (c). *The Solution for Small Values of q.*

For small values of q we may expand the integrand in powers of q, the analytical processes of expansion and integration term by term being easily justified.

We have then

$$\frac{\pi\sqrt{2\pi}}{Rgp} = \int_1^{\infty} (y-1)\, dy \int_{-1}^{+1} \frac{1}{\sqrt{1 - x^2}} (1 + q^2 a_2 + q^4 a_4 + \ldots) \exp\left(-\frac{y^2 p^2}{2} \right) dx$$

where .. .. (28)

$$\left.\begin{aligned} a_2 &= -\frac{x^2}{2} + \frac{p^2 x^2 y^2}{2} \\ a_4 &= \frac{x^4}{8} - \frac{p^2 x^4 y^2}{4} + \frac{p^4 x^4 y^4}{24} \\ a_6 &= -\frac{x^6}{48} + \frac{p^2 x^6 y^2}{16} - \frac{p^4 x^6 y^4}{48} + \frac{p^6 x^6 y^6}{720} \\ a_8 &= \frac{x^8}{384} - \frac{p^2 x^8 y^2}{96} + \frac{p^4 x^8 y^4}{192} - \frac{p^6 x^8 y^6}{1440} + \frac{p^8 x^8 y^8}{40{,}320} \end{aligned}\right\} \quad (29)$$

etc.

The odd powers of q vanish on integration with respect to x so are not calculated. It follows that

$$\frac{\pi\sqrt{2\pi}}{Rgp} = I_0 + q^2 I_2 + q^4 I_4 + q^6 I_6 + \ldots \quad .. \quad .. \quad (30)$$

where

$$\left.\begin{aligned} I_0 &= \pi\int_1^\infty (y-1)\exp(-\tfrac{1}{2}p^2y^2)\,dy \\ I_2 &= \frac{\pi}{4}\int_1^\infty (y-1)(-1+p^2y^2)\exp(-\tfrac{1}{2}p^2y^2)\,dy \\ I_4 &= \frac{\pi}{64}\int_1^\infty (y-1)(3-6p^2y^2+p^4y^4)\exp(-\tfrac{1}{2}p^2y^2)\,dy \\ I_6 &= \frac{\pi}{2304}\int_1^\infty (y-1)(-15+45p^2y^2-15p^4y^4+p^6y^6)\exp(-\tfrac{1}{2}p^2y^2)\,dy \end{aligned}\right\} \quad (31)$$

etc.

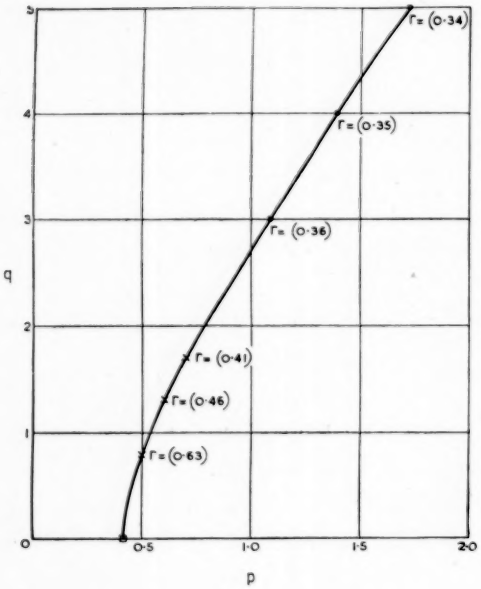

Fig. 5.

Now, if we put

$$l_n = \int_1^\infty y^n \exp(-\tfrac{1}{2}p^2y^2)\,dy \quad .. \quad (32)$$

and use the identity

$$\int_1^\infty y^{n+2}\exp(-\tfrac{1}{2}p^2y^2)\,dy = \frac{n+1}{p^2}\int_1^\infty y^n\exp(-\tfrac{1}{2}p^2y^2)\,dy + \frac{1}{p^2}\exp(-\frac{p^2}{2}) \quad .. \quad .. \quad (33)$$

we find

$$\left.\begin{aligned} I_0 &= \pi(l_1 - l_0) \\ I_2 &= \frac{\pi}{4}(l_1) \\ I_4 &= \frac{\pi}{16}(p^2l_1 - l_1) \\ I_6 &= \frac{\pi}{2304}(p^4l_1 - 6p^2l_1 + 3l_1) \\ I_8 &= \frac{\pi}{147{,}456}(p^6l_1 - 13p^4l_1 + 45p^2l_1 - 15\,l_1) \end{aligned}\right\} \quad (34)$$

etc.,

where

$$l_0 = \frac{1}{p}\sqrt{\frac{\pi}{2}}\left\{1 - \operatorname{erf}\left(\frac{p}{\sqrt{2}}\right)\right\} \quad .. \quad (35)$$

$$l_1 = \frac{1}{p^2}\exp(-\frac{p^2}{2}) \quad .. \quad .. \quad (36)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\{-\eta^2\}\,d\eta \quad .. \quad (37)$$

It follows at once that

$$1 + \tfrac{1}{4}q^2 + \theta_4q^4 + \theta_6q^6 + \theta_8q^8 + \ldots = \frac{p\sqrt{2}}{\phi\left(\frac{p}{\sqrt{2}}\right)}\left\{\frac{2}{Rg} + 1 - \operatorname{erf}\left(\frac{p}{\sqrt{2}}\right)\right\} \quad (38)$$

where

$$\left.\begin{aligned} \theta_4 &= \frac{p^2-1}{64} \\ \theta_6 &= \frac{p^4-6p^2+3}{2304} \\ \theta_8 &= \frac{p^6-15p^4+45p^2-15}{147{,}456} \\ \phi(x) &= \frac{2}{\sqrt{\pi}}\exp(-x^2) \end{aligned}\right\} \quad (39)$$

For a given value of p equation (38) becomes a power series in q with constant coefficients. It is easy to obtain sufficiently approximate values of q satisfying this for values of p less than 0.8. The values obtained for $Rg = 2$ are shown in Table I.

TABLE I.

p	q	r
0.5	0.8	0.625
0.6	1.3	0.460
0.7	1.7	0.412

These values are shown plotted as crosses in Fig. 5. Unfortunately for larger values of p the series in equation (38) converges

slowly and although it is easy to obtain other terms in the series the labour of calculating increases rapidly with increasing p. For large values of q it therefore becomes necessary to develop another expansion.

4 (d). *The Solution for Large Values of q.*

As before, we have

$$\frac{\pi\sqrt{2\pi}}{Rgp} = \int_1^\infty (y-1)\,dy \int_{-1}^{+1} \frac{1}{\sqrt{1-p^2}} \exp\left\{-\frac{q^2}{2}(yr-x)^2\right\} dx \quad (23)$$

For large values of q, the main contribution to the integral will come from small values of $(yr - x)$. We therefore make the transformation

$$\left.\begin{aligned} ry - x &= s \\ x &= t \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (40)$$

and seek a solution for small values of s.

We have

$$\frac{\pi\sqrt{2\pi}}{Rgp} = \frac{1}{r^2}\int_{(r-1)}^{(r+1)} (s+t+r)\,ds \int_{r-s}^{+1} \frac{1}{\sqrt{1-t^2}} \exp\left(-\frac{q^2s^2}{2}\right) dt + \frac{1}{r^2}\int_{(1+r)}^{\infty} (s+t-r)\,ds \int_{-1}^{+1} \frac{1}{\sqrt{1-t^2}} \exp\left(-\frac{q^2s^2}{2}\right) dt \quad .. \quad (41)$$

whence

$$\left.\begin{aligned} \frac{r^2\pi\sqrt{2\pi}}{Rgp} &= \pi\int_{1+r}^{\infty} (s-r)\exp\left(-\frac{q^2s^2}{2}\right) ds \\ &+ \int_{r-1}^{r+1} (s-1)\left\{\frac{\pi}{2} - \sin^{-1}(r-s)\right\} \exp\left(-\frac{q^2s^2}{2}\right) ds \\ &+ \int_{r-1}^{+} \sqrt{1-(r-s)^2} \exp\left(-\frac{q^2s^2}{2}\right) ds \end{aligned}\right\} \quad (42)$$

If we put $\frac{qs}{\sqrt{2}} = \phi$ and use the expansion

$$\sqrt{1-x^2} + x\sin^{-1}x = 1 + \tfrac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{80}x^6 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (43)$$

we obtain

$$\left.\begin{aligned} \frac{r^2\pi\sqrt{2\pi}}{Rgp} &= \pi\left(\frac{\sqrt{2}}{q}\right)\int_\beta^\infty \phi\exp(-\phi^2)\,d\phi \\ &- \pi\frac{\sqrt{2}}{q}r\int_\beta^\infty \exp(-\phi^2)\,d\phi \\ &- b_0\frac{\sqrt{2}}{q}\left(R_0 - \frac{\pi r}{2}\right) \\ &+ b_1\left(\frac{\sqrt{2}}{q}\right)^2(R_1 + \pi/2) \\ &- b_2\left(\frac{\sqrt{2}}{q}\right)^3 R_2 + b_3\left(\frac{\sqrt{2}}{q}\right)^4 R_3 \\ &- b_4\left(\frac{\sqrt{2}}{q}\right)^5 R_4 + \ldots . \end{aligned}\right\} \quad (44)$$

where

$$b_n = \int_\alpha^\beta \phi^n \exp\{-\phi^2\}\,d\phi; \quad \alpha = \frac{q(r-1)}{\sqrt{2}}; \quad \beta = \frac{q(r+1)}{\sqrt{2}} \quad .. \quad .. \quad (45)$$

and

$$\left.\begin{aligned} R_0 &= 1 + \tfrac{1}{2}r^2 + \tfrac{1}{24}r^4 + \tfrac{1}{80}r^6 \\ R_1 &= r + \tfrac{1}{6}r^3 + \tfrac{3}{40}r^5 + \ldots \\ R_2 &= \tfrac{1}{2} + \tfrac{1}{4}r^2 + \tfrac{3}{16}r^4 + \ldots \\ R_3 &= \tfrac{1}{6}r + \tfrac{1}{4}r^3 + \ldots \end{aligned}\right\} \quad (46)$$

In the expansion of the third integral on the right-hand side of equation (42) we have retained powers of s up to the fourth. It is quite easy to use the higher terms to obtain solutions for smaller values of q, but this approximation is sufficient for our purpose.

We now use the identity

$$b_{n+2} = \frac{n+1}{2}b_n - \tfrac{1}{2}(\beta^{n+1}\exp\{-\beta^2\} - \alpha^{n+1}\exp\{-\alpha^2\}) \quad .. \quad .. \quad (47)$$

and the approximation (valid for large positive values of y)

$$\int_y^\infty \exp(-x^2)\,dx = \frac{1}{2y}\left\{1 - \frac{1}{2y^2} + \frac{1.3}{(2y^2)^2} - \ldots\right\}\exp(-y^2) \quad .. \quad .. \quad (48)$$

It should be noted that for large values of q, r will be less than unity so that α will

be negative. It is easily seen that

$$b_0 = \sqrt{\pi} + \left(\frac{1}{2\alpha} - \frac{1}{4\alpha^3}\right)\exp(-\alpha^2) - \left(\frac{1}{2\beta} - \frac{1}{4\beta^3}\right)\exp(-\beta^2) \quad .. \quad .. \quad (49)$$

All the terms involving the exponential will be small compared with unity so that the large terms on the right-hand side of equation (44) come from b_0.

We obtain

$$\left.\begin{aligned} &\frac{\sqrt{2\pi}}{r}\left[R_0 + \frac{1}{q^2}R_2 + \frac{3}{q^4}R_4 + \ldots\right] \\ &+ \frac{1}{r}\cdot\frac{1}{q}\exp\left\{-\frac{q^2(r-1)^2}{2}\right\}\left[-\frac{\pi}{2}\frac{1}{(r-1)} + \frac{R_0}{(r-1)} - R_1 + R_2(r-1) - \ldots\right] \\ &+ \frac{1}{r}\cdot\frac{1}{q}\exp\left\{-\frac{q^2(r+1)^2}{2}\right\}\left[-\frac{\pi}{2}\cdot\frac{(2r+1)}{2(r+1)} - \frac{R_0}{(r+1)} + R_1 - R_2(r+1) + \ldots\right] \\ &= \sqrt{2\pi}\left[\frac{\pi}{Rg} + \frac{\pi}{2}\right] \end{aligned}\right\} \quad (50)$$

The higher order terms on the left-hand side of equation (50) are easy to obtain by retaining additional terms in equation (44) and will be of the form

$$\frac{1}{r}\cdot\frac{1}{q^n}\exp\left\{-\frac{q^2}{2}(r-1)^2\right\}F_n(r, R_0, R_1 \ldots)$$

and

$$\frac{1}{r}\cdot\frac{1}{q^n}\exp\left\{-\frac{q^2}{2}(r+1)^2\right\}G_n(r, R_0, R_1 \ldots) \quad .. \quad .. \quad (51)$$

The approximation in equation (50) gives sufficiently accurate results for values of q not less than 3. For given Rg the right-hand side of equation (50) is a constant and the equation is of the form

$$F(r, q) = \text{constant} \quad .. \quad .. \quad (52)$$

For given q it is therefore a straightforward process to obtain the corresponding value of r and therefore of p. The results are given in Table II. These values are plotted as circles in Fig. 5. The curve passes through the point $q = 0$, $p = 0.42$ from equation (15). The values of r corresponding to the various values of p and q are plotted along the curve, and for large values of p and q the value of r becomes asymptotically equal to 0.33 from equation (27).

TABLE II.

q	r	p
5	0.34	1.72
4	0.35	1.39
3	0.36	1.09

It is clear from Fig. 5 that if $(\bar{v}_2)_a$ is the output reading due to a noise input of R.M.S. value σ with no signal and $(\bar{v}_2)_b$ is the output due to a noise input with R.M.S. value σ plus a sinusoidal signal with R.M.S. value $c/\sqrt{2}$ equal to σ, then $q = \sqrt{2}$ and $p = 0.63$. Thus

$$(\bar{v}_2)_b/(\bar{v}_2)_a = 1.5$$

and the error involved in assuming that the response of the linear rectifier to signal plus noise is increased by a factor $\sqrt{2}$ over that due to the noise alone (as in the case of a thermo-junction) is about 6 per cent. or 0.5 db.

This figure is of course true only when the input R.M.S. noise is equal to the input R.M.S. signal. For larger ratios of signal/noise input the corresponding error is larger, for example if the input signal power is twice the noise input power the error becomes 0.6 db.

5. Formula for Noise-Modulated Carrier working on Exponential Part of Diode Characteristic

In the circuit of Fig. 1 the capacitor is assumed so large that the voltage $\bar{v}_2$ across R is substantially constant and equal to $\bar{v}_2$. The characteristic of the diode now is taken to be :—

$$i = A\exp\{b(v_1 - \bar{v}_2)\} \quad .. \quad .. \quad (53)$$

where

$$\left.\begin{aligned} A &= 10^{-4} \text{ amperes} \\ b &= 10 \text{ (volts)}^{-1} \end{aligned}\right\} \quad .. \quad .. \quad (54)$$

This characteristic can be assumed to hold up to about 100 μA. Between some 100 μA and 200 μA there is a gradual change-over from the exponential characteristic to the linear or three-halves power law.

Now, as in Section 4, if the input voltage consists of a sinusoidal signal with amplitude c and frequency $\omega/2\pi$ together with noise

whose amplitude distribution is given by

$$W(v_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{v_1^2}{2\sigma^2}\right\} \quad .. \quad (55)$$

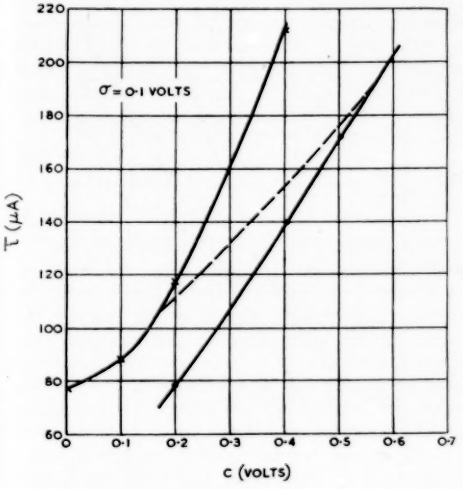

Fig. 6.

we have

$$\bar{\imath} = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} A \exp\{b(v_1 - \bar{v}_2)\}\, dv_1 \int_{-1}^{+1} \frac{1}{\sqrt{1-y^2}} \exp\left\{-\frac{(v_1 - cy)^2}{2\sigma^2}\right\} dy \quad .. \quad .. \quad (56)$$

Here, in view of the fact that we have used $+\infty$ for the upper limit of v_1, we assume that either the exponential characteristic holds for all values of v_1, or that the input voltage is limited in value to the exponential part of the characteristic. Neither assumption is, in fact, true; so long as the input involves noise with the distribution (55), theoretically infinite amplitudes will occur, but so long as the mean diode current is less than that at which the exponential characteristic ceases to be valid, it may be safely assumed that the high amplitude noise peaks occur sufficiently rarely to allow (56) to be a good approximation.

From (56) it follows that:

$$\bar{\imath} = \bar{v}_2/R = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} A \exp\{b(v_1 - \bar{v}_2)\} dv_1 \int_{-1}^{+1} \frac{1}{\sqrt{1-y^2}} \exp\left\{-\frac{(v_1 - cy)^2}{2\sigma^2}\right\} dy \quad .. \quad .. \quad (57)$$

From which, by reversing the order of integration, we obtain

$$\bar{v}_2 \exp(b - \bar{v}_2) = RA \exp(\tfrac{1}{2}\sigma^2 b^2)\, J(jbc) \quad .. \quad .. \quad (58)$$

with the usual Bessel Function Notation.

5 (a). *The Solution for Small Values of the Load Resistance.*

If R is small, then σ and c must be small in order that the diode may operate on the exponential part of its characteristic.

The results cannot be conveniently plotted in the form of a relation between p and q as was done in Section 4, since the relation between p and q for the exponential case is not independent of σ so that for each value of σ there is a different $p-q$ curve. This method of plotting the results therefore loses its usefulness. In Figs. 6, 7 and 8 are

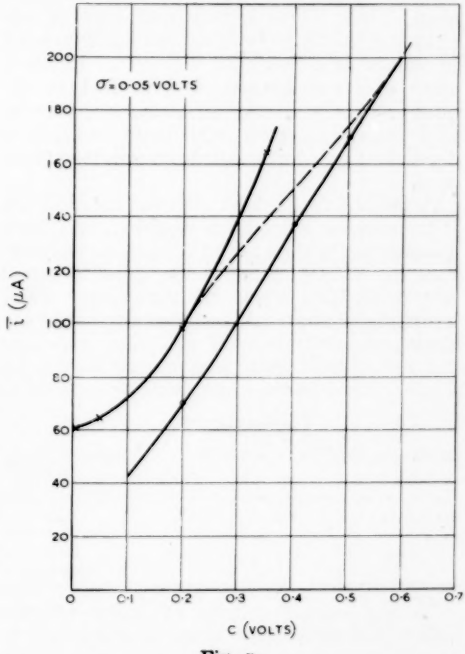

Fig. 7.

plotted as crosses results from equation (58) with $R = 1000$ ohms connecting $\bar{\imath}$ and c. The curves in these figures with circles are

the results obtained for the various values of σ assuming the linear diode characteristic and the formulae derived in Section 4. From somewhere below 150 μA on the exponential-diode curve a transition to the linear diode curve takes place, the line joining the latter somewhere above 150 μA. Dotted lines are drawn suggesting a possible transition characteristic, although it is likely that those shown are not sufficiently gradual.

These results show that, with a small load resistance, measurement of noise of small amplitude using the exponential part of the diode characteristic is not practicable. For values of σ between about 0.1 volt and 1 volt the transition part of the characteristic and the linear part are mainly involved and this range requires additional analysis although it is certain that the same difficulties which are mentioned below for smaller values of σ will occur for this range also. Thus if the exponential part of the characteristic is to be used, we are restricted to values of σ less than 0.1 volt. Further, since σ is not known, and in fact is the quantity to be measured, it is necessary to find the range over which the output is dependent on c but substantially independent of σ.

A method often used with the linear diode is to increase c until $\bar{\imath}$ is doubled, and then assume (as in the case of a thermo-junction) that the R.M.S. value of the signal is equal to the R.M.S. value of the noise. That is certainly not the case here as Table III shows.

TABLE III.

σ	$\bar{\imath}$ (μA) ($c = o$)	c_1	$\frac{c_1}{\sigma}$
0.1	77	0.40	4.0
0.05	60	0.27	5.4
0.01	57	0.28	30.0 (approx.)

In the second column of Table III the value of $\bar{\imath}$ with noise alone is given and in the third column c_1 (the value of c which doubles the current). It is seen that the ratio c_1/σ is by no means constant and for small values of σ the increase is extremely rapid. In addition to this, these values involve the use of the transition part of the characteristic which is not known at all accurately.

The fact that with the exponential characteristic the rectified output current bears little direct relation to the R.M.S. input with a noise modulated signal means that it is necessary to be certain that the noise

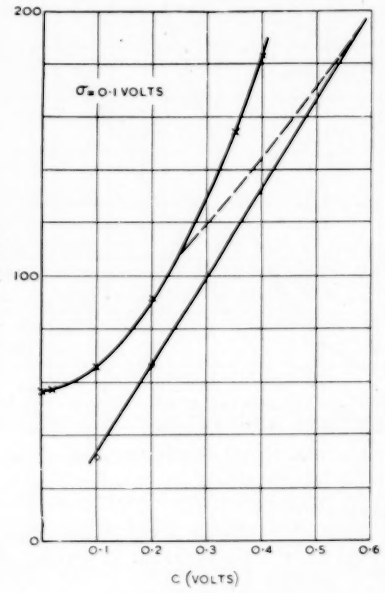

Fig. 8.

amplitude is large enough to make the exponential part of the characteristic unimportant when making noise measurements with a diode.

5 (b). *The Solution for Large Values of the Load Resistance*

If the load resistance is large compared with the diode impedance, larger values of the input voltage are necessary in order to obtain an appreciable rectified output current. For values of σ and c greater than about 1 volt an approximate analytical solution of Equation (58) can be obtained. Using the asymptotic approximation for $J(jbc)$

$$J(jbc) \doteq \frac{\exp(jbc)}{\sqrt{2\pi jbc}} \qquad .. \quad .. \quad (59)$$

which involves not more than 1 per cent. error if $bc > 10$; we have

$$b\bar{v}_2 \exp\left(b\bar{v}_2 - \tfrac{1}{2}\sigma^2 b^2 - bc\right) - \frac{bRA}{\sqrt{2\pi bc}} = 0$$
$$.. \quad .. \quad (60)$$

Now, if we take for example $R = 10^6$ ohms, since $b\bar{v}_2$ will be >50 and the second term in equation (60) will be of the order of 100, a variation in $b\bar{v}_2 - \frac{1}{2}\sigma^2b^2 - bc$ of one or two units will always be sufficient to change the sign of the left-hand side of equation (60). But this will be only 1 per cent. or 2 per cent. in the value of $b\bar{v}_2$; so that an approximate solution of equation (60) is

$$b\bar{v}_2 = \tfrac{1}{2}\sigma^2b^2 + bc \quad .. \quad .. \quad (61)$$

that is

$$\bar{\imath} = 5\sigma^2 + c \quad .. \quad .. \quad .. \quad (62)$$

in microamperes, if $R = 10^6$ ohms.

The results derived in this case seem to be even less applicable to the measurement of noise than is the case for a small R. Curves are shown in Fig. 9 for $\sigma = 4$. Again the crosses refer to the exponential characteristic and the circles to the linear characteristic, the dotted curve being a possible transition from one to the other. A point of interest and one of the reasons why noise measurements cannot easily be made by this method is the large ratio c/σ required before the magnitude of the rectified output current is appreciably altered. This is due to the fact that the large R gives great weight to the effect of the large amplitude noise peaks and for this reason any limitation of amplitude due to saturation in the amplification of the noise will make calculations for large R inaccurate.

5 (c). *Very Small Values of the Load Resistance*

When σ is of the order of 1 mV it might be possible to work with R considerably less than 1,000 ohms. The transition region in this way can be avoided by keeping $\bar{\imath}$ down to the order of 50 μA and the value of c/σ for a given increase of $\bar{\imath}$ can be made more nearly independent of σ. The actual values for such a case are not worked out here, but there may in some cases be a practical value in measurements made in this way. Values are readily obtainable from equation (58).

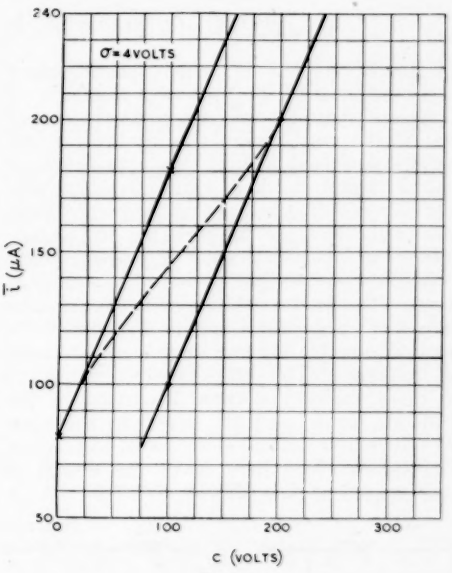

Fig. 9.

In conclusion we wish to tender our acknowledgments to The General Electric Co., Ltd. and The Marconiphone Co., Ltd. on whose behalf the work which has led to this publication was carried out.

REFERENCES

[1] See for example S. O. Rice, *Bell S. Tech. Journ.*, Vol. 23, July, 1944.
[2] W. R. Bennett, *Bell S. Tech. Journ.*, Vol. 23, p. 97, 1944

FILTER DESIGN TABLES BASED ON PREFERRED NUMBERS*

Low-pass Filters

By H. Jefferson, B.A., A.M.I.E.E., A.Inst.P.

IN another paper† the author has discussed the application of preferred numbers to filter design. It was shown in this paper that if the final design was to involve the use of preferred values of capacitance, a logical design method followed when preferred impedance and cut-off frequency were specified. The use of preferred impedance values is also desirable if commercially available resistors are to form the terminations. The tables which follow enable the values of capacitance and inductance to be read off directly for any impedance and cut-off frequency. The errors involved in the use of these tables are those discussed in the previous paper, and are less than the error introduced by normal commercial tolerances. The tables given here cover the design of constant-k low-pass filters.

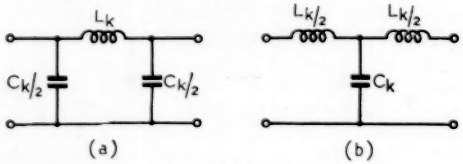

Fig. 1. *π- and T-filter sections.*

The tables give the values of the usual ladder network elements. When only single sections are used, the actual principal element values are those shown in Fig. 1, where L_k and C_k are the values given by the tables, and are approximations to

$$L_k = R/\pi f_c \quad \text{and} \quad C_k = 1/\pi f_c R$$

where R is the design impedance and f_c is the cut-off frequency.

It was shown in the earlier paper that preferred values for $X/2$ exist for all preferred values of X, and for convenience these are listed below.

The tables are in pairs, each pair being used to determine the value of one element of the filter. The first table of each pair is of coarse mesh, in half-decade (logarithmic) steps. From this the order of magnitude of the capacitance or inductance is found by examination of the values which border it. The second table covers only one decade and enables the significant figures to be found.

As an example of the use of the tables, let us consider a low-pass filter of 620 ohms impedance and 5,100 c/s cut-off frequency. From Table I, the capacitance is in the square bounded by 0.33 μF, 0.1 μF, 0.033 μF and 0.1 μF. From Table III the inductance is in the square bounded by 33 mH, 10 mH, 33 mH and 0.1 H. Table II gives the significant figures for the capacitance as 10, so that the actual value of capacitance is 0.10 μF. Table IV gives the significant figures for the inductance as 39, so that the actual inductance value is 39 mH. An exact computation gives $L_k = 38.4$ mH and $C_k = 0.1004$ μF, so that the error is less than 2 per cent. The physical form of a single section is shown in Fig. 2.

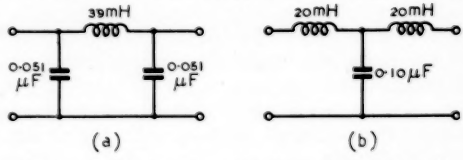

Fig. 2. *The component values of the example discussed in the text are marked on these π- and T-sections.*

PREFERRED VALUES OF $X/2$ FOR PREFERRED VALUES OF X.

X	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	100
$\frac{X}{2}$	5.1	5.6	6.2	6.8	7.5	8.2	9.1	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51

* MS. accepted by the Editor, July 1945.

† "Preferred Numbers and Filter Design": *Wireless Engineer*, October 1945.

TABLE I

CAPACITANCE C_k OF LOW-PASS FILTER

DESIGN IMPEDANCE (OHMS)

CUT-OFF FREQUENCY	1	3.3	10	33	100	330	1 000	3 300	10 000	33 000	100 000	330 000
1	3.3×10^{5}	1×10^{5}	3.3×10^{4}	1×10^{4}	3.3×10^{3}	1×10^{3}	330	100	33	10	3.3	1.0
3.3	1×10^{5}	3.3×10^{4}	1×10^{4}	3.3×10^{3}	1×10^{3}	330	100	33	10	3.3	1.0	0.33
10	3.3×10^{4}	1×10^{4}	3.3×10^{3}	1×10^{3}	330	100	33	10	3.3	1.0	0.33	0.1
33	1×10^{4}	3.3×10^{3}	1×10^{3}	330	100	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}
100	3.3×10^{3}	1×10^{3}	330	100	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}
330	1×10^{3}	330	100	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}
1 000	330	100	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}
3 300	100	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}	330
10 000	33	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}	330	100
33 000	10	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}	330	100	33
100 000	3.3	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}	330	100	33	10
330 000	1.0	0.33	0.1	3.3×10^{-2}	1×10^{-2}	3.3×10^{-3}	1×10^{-3}	330	100	33	10	3.3
c/s	μF							$\mu\mu$F				
M/cs	$\mu\mu$F							$\mu\mu\mu$F				

TABLE II

SIGNIFICANT FIGURES FOR C_k OF LOW-PASS FILTER

DESIGN IMPEDANCE

CUT-OFF FREQUENCY	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10
10	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33
11	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	65	62	56	51	47	43	39	36	33	30
12	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27
13	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24
15	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22
16	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20
18	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18
20	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16
22	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15
24	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13
27	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12
30	11	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11
33	10	91	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10
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39	82	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82
43	75	68	62	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75
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56	56	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56
62	51	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51
68	47	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47
75	43	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43
82	39	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39
91	36	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36
10	33	30	27	24	22	20	18	16	15	13	12	11	10	91	82	75	68	62	56	51	47	43	39	36	33

TABLE III

INDUCTANCE L_k OF LOW-PASS FILTER

DESIGN IMPEDANCE (OHMS)

CUT-OFF FREQUENCY	1	3.3	10	33	100	330	1000	3300	10 000	33 000	100 000	330 000
1	0.33	1.0	3.3	10	33	100	330	1000	3300	10 000	33 000	100 000
3.3	0.10	0.33	1.0	3.3	10	33	100	330	1000	3300	10 000	33 000
10	33	0.10	0.33	1.0	3.3	10	33	100	330	1000	3300	10 000
33	10	33	0.10	0.33	1.0	3.3	10	33	100	330	1000	3300
100	3.3	10	33	0.10	0.33	1.0	3.3	10	33	100	330	1000
330	1.0	3.3	10	33	0.10	0.33	1.0	3.3	10	33	100	330
1000	0.33	1.0	3.3	10	33	0.10	0.33	1.0	3.3	10	33	100
3300	0.10	0.33	1.0	3.3	10	33	0.10	0.33	1.0	3.3	10	33
10 000	33	0.10	0.33	1.0	3.3	10	33	0.10	0.33	1.0	3.3	10
33 000	10	33	0.10	0.33	1.0	3.3	10	33	0.10	0.33	1.0	3.3
100 000	3.3	10	33	0.10	0.33	1.0	3.3	10	33	0.10	0.33	1 0
330 000	1.0	3.3	10	33	0.10	0.33	1.0	3.3	10	33	0.10	0.33
c/s	μH				mH				H			
kc/s	μH x 10^{-3}				μH				mH			
Mc/s	$\mu\mu$H				μH x 10^{-3}				μH			

TABLE IV

SIGNIFICANT FIGURES FOR L_k OF LOW-PASS FILTER

DESIGN IMPEDANCE

CUT-OFF FREQUENCY	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10
10	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33
11	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30
12	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27
13	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24
15	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22
16	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20
18	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18
20	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16
22	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15
24	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13
27	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12
30	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10	11
33	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91	10
36	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91
39	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82
43	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75
47	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68
51	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62
56	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56
62	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51
68	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47
75	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43
82	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39
91	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33	36
10	33	36	39	43	47	51	56	62	68	75	82	91	10	11	12	13	15	16	18	20	22	24	27	30	33

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Diode Input Resistance

To the Editor, "Wireless Engineer."

SIR,—In the October 1941 issue of *Wireless Engineer* you published a paper by D. A. Bell on the "Diode as Rectifier and Frequency-Changer," and I should like to make two comments on it.

Equation (14) for the shunt input resistance can be simplified to

$$\rho_0 = \frac{2\pi R_a}{\alpha - \sin\alpha}$$

using

$$\frac{R}{R_a} = \frac{\pi}{\tan\alpha/2 - \alpha/2}$$

These results for the linear detector had been derived some years previously by F. M. Colebrook.*

The "peculiar kinks" in Figs. 10, 11 and 12 are due to computational errors, since an accurate evaluation of the quantities graphed shows that they vary smoothly over the range of R_a/R from 0.5 to 0.9.

R. E. BURGESS,
Radio Division, N.P.L.

Pulse Modulation

To the Editor, "Wireless Engineer."

SIR,—The article by Roberts and Simmonds in your November issue dealing with the use of modulated pulses in communication systems can be expected to foreshadow the publication of further information regarding this comparatively new technique. Recently disclosed details of the Army's Wireless Set No. 10 reveal that pulse multiplex has already been put to useful effect, and there is now no doubt a wealth of knowledge available in various quarters, which having accumulated during the war years, is likely to be released in technical publications.

With the expectation of further contributions in the near future, and bearing in mind the greater variety of possible modulating systems resultant upon the use of discrete pulses, there is a need and opportunity for early rationalisation of nomenclature so as to avoid a potential source of confusion, which the writer knows all too well is apt to fog clarity of exposition and discussion in this field.

Doubtless several types of pulse modulation will emerge for discussion in the future, but since one may reasonably expect them to be related to the basic types described by Roberts and Simmonds, it is pertinent to examine critically the names with which they have been christened in this paper.

In the main the authors' choice appears to be a judicious one, and no one is likely to quarrel with their usage of the terms "pulse amplitude modulation" or "pulse frequency modulation," but it may be argued there is a case to prefer "pulse length modulation" to "pulse width modulation," since pulse length has a more precise meaning than pulse width when expressed in units of time, as well as being in conformity with current parlance in the U.S.A. and other quarters.

The term "pulse delay modulation" does, however, appear to be open to criticism on the grounds that the pulses convey the information by advancement in mean time of occurrence as well as by delay in mean time of occurrence. It is suggested that pulse phase modulation (PPM) is a more exact description, since the modulation, whether in advance or retard of the normal instant of occurrence is applied as a variation in the phase of occurrence of a recurrent wave form. Furthermore, the term then comes into line with the accepted description of a similar form of modulation when applied to a steady carrier, namely "phase modulation."

The authors draw a distinction between pulse frequency modulation and pulse number modulation. The reason for the distinction is not immediately clear, since the frequency of recurrence of a pulse train means precisely the number of pulses occurring per second, unless perhaps a system is envisaged in which modulation is accomplished by suppression or insertion of pulses in a recurrent pulse train. Such a system would not appear to be able to deal faithfully with a continuously varying modulating waveform.

It is highly probable that further comparisons will be drawn between pulse multiplex systems and carrier multiplex systems, and once again it would be useful if the same terminology could be used in future discussion. In referring to the two systems as "time allocation multiplex" and "frequency allocation multiplex" respectively, the authors have coined two terms which, at the same time as providing a nutshell description of the technique, also draw a basic distinction between them. One feels that these terms could be perpetuated with advantage.

D. COOKE.
Signals Research and Development Establishment, Ministry of Supply.

* F. M. Colebrook. "The Theory of the Straight-Line Rectifier," *Wireless Engineer*, 1930, Vol. 7, pp. 595–603.

The Physical Society's Exhibition

THE 30th Exhibition of Scientific Instruments and Apparatus will be held at the Imperial College of Science and Technology, Imperial Institute Road, South Kensington, London, S.W.7. It will be open for three days, 1st Jan., 2.30–9 p.m.; 2nd Jan., 4–9 p.m., and 3rd Jan., 2.30–9 p.m. Admission is by ticket only. Tickets may be obtained from the Exhibition Secretary, 1, Lowther Gardens, Exhibition Road, London, S.W.7, but members of Institutions and Scientific Societies may obtain them from their Secretaries.

In addition to the Exhibition there will be discourses; on 1st Jan., at 5.30 and 8.15 p.m., "The Optical Industry in the War," by Captain R. Martin, M.Sc.; on 2nd Jan., at the same times, "Radar," by Sir Edward Appleton, K.C.B., F.R.S.; and, on 3rd Jan., at 5.30 and 9 p.m., "Modern Plastics and Cements," by J. C. Swallow, Ph.D., B.Sc.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

570 337.—Design and construction of a moving-coil microphone wherein various spaces surrounding the elements are utilised to secure an improved frequency characteristic.

Standard Telephones and Cables Ltd. and M. L. Gayford. Application date 28th September, 1943.

570 370.—Holder or casing for a piezo-electric pick-up having sufficient flexibility to accommodate crystals of slightly different sizes.

The Brush Development Co. (assignees of H. B. Shapiro). Convention date (U.S.A.) 26th March, 1942.

AERIALS AND AERIAL SYSTEMS

570 568.—Auxiliary reflector for further concentrating the beam in an aerial of the parabolic mirror type.

Cie. Generale de Telegraphie sans Fil. Convention date (France) 15th June, 1940.

570 888.—Kite or balloon aerial which is also adapted to supply lighting current to a suspended morsing or signal lamp.

International Marine Radio Co. Ltd.; H. Thorpe-Woods and D. H. Harrison. Application date 21st June, 1944.

DIRECTIONAL WIRELESS

570 038.—Directional system in which the dipole of a parabolic reflector is energised by a wave-guide feeder.

The General Electric Co. Ltd. and R. J. Clayton. Application date 7th January 1942.

570 201.—Directive system of the overlapping-beam type in which at least two directly-energised aerials are used in co-operation with side reflectors.

Standard Telephones and Cables Ltd. and L. J. Heaton-Armstrong. Application date 12th January, 1942.

570 203.—Phase-shifting devices, particularly for direction-finding, or for radiating a directable beam of energy.

H. Jefferson. Application date 4th February, 1942.

570 204.—Direction-finding system in which the analysis of Morse signals is facilitated by the use of a pair of cathode-ray indicators.

L. J. van Rooyen and W. O. Agar. Application date 4th February, 1942.

570 566.—Preventing errors, due to reflection from the different stratospheric layers, in a direction-finder of the cathode-ray type.

H. Jefferson. Application date 26th January, 1942.

570 986.—Device geared to the rotary coil of a directional aerial for indicating the bearing as a spot of light on a translucent map. (Addition to 551 376.)

J. Thorley. Application date 2nd March, 1943.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

570 008.—Frequency-converting circuit for a super-heterodyne receiver in which the I.F. oscillations are maintained at constant intensity, though a variable feed-back is used to increase the selectivity of the set.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 25th September, 1942.

570 092.—Midget receiver designed to be plugged as a unit into any electric lamp-holder or wall socket.

F. M. Hills. Application date 5th November, 1943.

570 224.—Tuning device in which a "logging" scale-indicator is mounted edge on to the main scale, and is geared down to move one division for a complete rotation of the latter.

Marconi's W.T. Co. Ltd., G. L. Grisdale and H. C. Norwood. Application date 9th December, 1943.

570 390.—Wireless receiver in which the frequency sweep of a local oscillator is synchronised with the time base of a cathode-ray tube in order to indicate, at any moment, the number of stations operating within a selected frequency range.

H. Jefferson. Application date 4th February, 1942.

570 391.—Receiver for indicating and identifying the various stations that are transmitting, at any time, within a selected frequency range.

H. Jefferson. Application date 4th February, 1942.

570 392.—Oscillation generator, with a highly-sensitive frequency control, suitable for a receiver designed to indicate, at any moment, the number of short-wave stations operating within a selected frequency range.

D. J. Fewings. Application date 4th February, 1942.

570 466.—Limiting the effect of static and like impulsive "noise" by a method which differentiates between positive and negative peaks of modulation.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 5th October, 1942.

570 474.—Variable tuning device comprising an inductance coil partly covered by a metal cylinder, so that the covered windings simulate a pair of Lecher wires.

"Patelhold" Patentverwertungs & Elektro-Holding A.G. Convention date (Switzerland) 17th December, 1942.

570 478.—Demodulating F.M. signals by using suitably-terminated wave-filters, in combination with a reactance to produce the necessary quadrature-voltage.

R. O. Carter. Application date 23rd December, 1943.

570 636.—Method of precision tuning to centimetre waves by utilizing the harmonics of a stabilised piezo-electric oscillator.

Marconi's W.T. Co. Ltd. and C. P. Beanland. Application date 22nd October, 1943.

570 772.—Receiver primarily designed to receive frequency-modulated signals, but adapted to be switched over to amplitude-modulated or C.W. signals.

The Mullard Radio Valve Co. Ltd. and C. L. Richards. Application date 30th July, 1943.

570 877.—Mass production of wireless sets by spraying or electroplating over a chassis which includes the forms of certain of the component parts and also the connecting leads.

J. A. Sargrove. Application date 30th August, 1943.

571 005.—Variable tuning inductance comprising two movable and one fixed winding, the latter being of opposite polarity to the first two, for giving a linear response over a wide tuning-range.

The British Thomson-Houston Co. Ltd. (communicated by the General Electric Co.). Application date 11th October, 1943.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

571 061.—Stereoscopic television in which right and left eye pictures are respectively focused upon the correspondingly-inclined ridges of a projection screen.

F. W. MacD. Matthews. Application date 20th September, 1943.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

570 094.—Series arrangement of a hard valve and a gas-filled valve to form a switching circuit, particularly for telegraph keying on mark-and-space signals.

Marconi's W.T. Co. Ltd. and L. C. Styles. Application date 10th November, 1943.

570 136.—Coaxial transmission line in which the insulating discs are varied logarithmically, in thickness or spacing, in order to control the frequency characteristic.

The British-Thomson Houston Co. Ltd. Convention date (U.S.A.) 8th March, 1940.

570 279.—Means for measuring the percentage modulation of a signal-carrying wave by passing the latter through two rectifiers in series.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 23rd November, 1942.

570 283.—Combined radio transmitter and receiver in which a single valve serves to control the frequency both of the incoming and outgoing signals.

Communications Patents Ltd. and G. B. Ringham. Application date 1st December, 1943.

570 339.—Terminals or couplings for coaxial transmission lines, in which a graded screw thread is used to ensure a readily-locked joint.

British Insulated Cables Ltd., H. R. F. Carsten and C. H. M. Thorpe. Application date 28th September, 1943.

570 359.—Means for preventing a pair of Lecher wires that are enclosed by a screen from resonating as a coaxial line instead of as a balanced pair of conductors.

Standard Telephones and Cables Ltd. and C. N. Smyth. Application date 21st June, 1940

570 379.—H.F. cable in which a central conductor, or a pair of spaced conductors, is enclosed in a polythene insulator on which a copper covering is deposited electrolytically.

Callender's Cables and Construction Co. Ltd., Callender-Suchy Developments Ltd., L. G. Brazier, R. McL. Fairfield and C. T. Suchy. Application date 22nd July, 1943.

570 415.—Radio signalling system with automatic dialling wherein the individual conversing frequency of a particular station is used for calling that station.

J. G. Murdoch and Co., Ltd. D. L. Clay and F. L. Hogg. Application date 4th December, 1943.

570 679.—Water-cooled valve in which a steep temperature-gradient is maintained between the anode of the valve and the cooling-liquid, in order to prevent the latter from boiling.

Standard Telephones and Cables Ltd. (assignees of C. V. Little). Convention date (U.S.A.) 5th October, 1942.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

570 488.—System for the secret transmission of speech in which selected frequency-bands are subjected to different attenuation.

"Patel Hold" Patentverwertungs & Elektro-Holding A. G. Convention date (Switzerland) 20th December, 1941.

571 214.—Duplex system of wired-wireless communication over existing telephone or power-supply lines, in which the same carrier-frequency is used both for incoming and outgoing messages.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 11th July, 1942.

571 279.—Generating infra-red rays for signalling purposes by using a self-extinguishing arc and a "glower" or non-ohmic resistance, such as that known under the trade-name "Insulcon."

Stupakoff Ceramic and Manufacturing Co. Convention date (U.S.A.) 19th March, 1942.

571 627.—Modulation system in which the carrier-wave is radiated in pulses of constant amplitude but of varying duration, the pulses being separated by intervals of zero radiation.

Westinghouse Electric International Co. Convention date (U.S.A.) 22nd October, 1942.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

570 213.—Construction and arrangement of the deflecting plates or electrodes in a cathode-ray tube.

O. E. H. Klemperer (commonly known as O. Klemperer). Application date 13th February, 1943.

570 468.—Cathode made in the form of a close-set cage of parallel bars, to give an increased surface of emission when indirectly heated.

Standard Telephones and Cables Ltd. (assignees of P. G. Chevigny). Convention date (U.S.A.) 8th December, 1942.

570 553.—Photo-electric cell wherein the photo-sensitive material is evaporated from the anode

on to the cathode, a movable shield screens the rest of the cell from undesired deposition.

Cinema-Television Ltd., G. A. R. Tomes and W. Blackman. Application date 5th July, 1941.

570 601.—Photo-electric cell with two sensitive electrodes each of which acts alternately as cathode and anode.

The Mullard Radio Valve Co. Ltd. and R. W. Kersey. Application date 6th September, 1943.

570 672.—Electrostatic screening to prevent interaction between the two pairs of deflecting plates in a cathode-ray tube, particularly for ultra-high frequencies.

G. Liebmann. Application date 19th August, 1943.

570 834.—Heat-expansion method of maintaining a pre-determined small clearance between the cathode and grid of a short-wave valve, say a grounded-grid triode.

G. Liebmann and Cathodeon Ltd. Application date 20th November, 1941.

SUBSIDIARY APPARATUS AND MATERIALS

569 700.—Electrode arrangement for a multiple-unit condenser of the rolled or layered type.

J. H. Cozens and The Telegraph Condenser Co., Ltd. Application date 3rd September, 1943.

569 858.—Generating periodic wave-trains at supersonic frequency, and their control for producing a stationary pattern on the screen of a C.R. tube, for detecting flaws in the structure of materials.

United Aircraft Corpn. Convention date (U.S.A.) 29th June, 1942.

569 995.—Screening arrangement for preventing radiation from the sparking plug of an internal combustion engine.

Titeflex Metal Hose Co. Convention date (U.S.A.) 9th October, 1942.

570 087.—Construction of transmission lines used as tuning elements, where it is desirable that the geometrical length should be less than the electrical length.

The General Electric Co. Ltd. and C. R. Dunham. Application date 26th May, 1943.

570 102.—Photo-electric relay, controlled by a gravity-operated screen, and serving to maintain a suspended system in a stable position.

Vickers-Armstrong Ltd., J. C. Clifton and F. W. Rabarts. Application dates 22nd February, 1943, *and 18th January,* 1944.

570 162.—Grid-controlled rectifier so coupled to an A.C. supply as to maintain a constant-voltage output, irrespective of fluctuations in the supply.

The British Thomson-Houston Co. Ltd., J. Moir and W. S. Graff-Baker. Application date 31st December, 1943.

570 188.—Fixed condenser constructed with inductive elements to serve as a generator of high-frequency impulses.

H. G. Solomon. Application date 9th July, 1943.

570 199.—Photo-electric cell in which a retarding electrode is provided to give the cell a logarithmic increase of sensitivity with rise of anode voltage.

W. Blackman, Cinema-Television Ltd. and G. A. R. Tomes. Application date 20th May, 1941.

570 255.—Screw-down link or terminal, as used on an accumulator, with a spring control for preventing accidental contacts.

Standard Telephones and Cables Ltd. and L. H. Webb. Application date 6th August, 1943.

570 299.—Voltage-stabilising device in which the grid of the main rectifier valve is controlled by a circuit which is shunted across the load and includes a hard valve in series with a glow-discharge tube.

The British Thomson-Houston Co. Ltd. and J. Moir. Application date 6th September, 1943.

570 346.—Method of making and processing the selenium elements used in rectifiers and photo-electric cells.

Standard Telephones and Cables Ltd. (assignees of A. J. Miller and E. P. Sauerborn). Convention date (U.S.A.) 25th November, 1942.

570 347.—Method of making a number of selenium elements in one jig-assembly.

Standard Telephones and Cables Ltd. (assignees of M. F. Skinner). Convention date (U.S.A.) 7th January, 1943.

570 385.—Jigging process for making selenium elements in bulk.

Standard Telephones and Cables Ltd. (assignees of P. R. Kallmeyer). Convention date (U.S.A.) 18th November, 1942.

570 559.—Use of nitro-benzene-sulphuric acid as an electrolyte for accumulators, particularly when the lead plates are sulphated.

Marconi's W.T. Co. Ltd. and C. P. Fagan. Application date 9th September, 1941.

570 717.—Process for coating the glass covers of indicating instruments with fluosilicic acid in order to reduce surface reflection.

Radio Corporation of America. Convention date (U.S.A.) 28th May, 1943.

570 722.—Method of mounting the active elements on a rectifier of the selenium type (addition to 526 482).

Standard Telephones and Cables Ltd. and E. A. Richards. Application date 15th January, 1944.

570 730.—Method of winding the magnetic core of a transformer from a ribbon of sheet steel having the crystal grains of the metal orientated in the direction of rolling.

G. R. Shepherd (communicated by Westinghouse Electric International Co.). Application date 20th January, 1944.

570 773.—Construction and assembly, around the ignition system of an internal combustion engine, of a screen to prevent H.F. radiation.

E. L. W. Byrne (communicated by Titeflex Metal Hose Co.). Application date 24th December, 1943.

571 000.—Robust terminal assembly and support for a ceramic condenser of the tubular or cup-shape type.

F. C. Stephan. Application date 29th September, 1943.

571 101.—Sealing-disc, made in part of synthetic rubber, for a high-voltage capacitor.

R. Trist & Co., Ltd., and D. S. Prince. Application date 27th November, 1943.

571 117.—Unitary assembly of the component parts of a wave-filter circuit.

N. E. Brookes (communicated by Tobe Deutschmann Corp.). Application date 14th December, 1943.